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ADDING & SUBTRACTING POLYNOMIALS

In this lesson you will learn about:

- like and unlike terms
- addition and subtraction of terms
- inverses of polynomials
- addition and subtraction of polynomials

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KEEPING SKILLFUL

EXERCISE 1 Complete each of the following operations.

1. $(-6) \div (-2) =$

2. $(-6)(2) =$

3. $(+128) \div (-16) =$

4. $(-2)(3)(-1)(10) =$

5. $6 - 7 =$

6. $-7 - (-8) =$

7. $\dfrac{-8 - 7}{-2 + 5} =$

8. $\dfrac{(-8)(-15)}{(3)(-4)} =$

9. $-6 + 4 - 2 - 3 + 7 + 10 + 21 - 1 =$

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ALGEBRA VOCABULARY

In Lesson 17 you learned a lot of words used in algebra. In this lesson and the following lessons you will need to know these words.

Reference

If you have forgotten any of the definitions, read Lesson 17 or page 207 of *Holt Mathematics 3*.

SELF-CORRECTING EXERCISE A

Write a short definition for each of the following words or phrases.

1. Algebraic expression _____

2. Polynomial _____

3. Terms _____

SELF-CORRECTING EXERCISE A (continued)

4. Factors _____

5. Constants _____

6. Variables _____

7. Literal coefficient _____

8. Numerical coefficient _____

9. Monomial _____

_____

10. Binomial _____

_____

11. Trinomial _____

_____

Check your definitions with those on page 21 of this lesson. *Remember that a definition in your own words is best, as long as the meaning is the same.*

POLYNOMIALS

Since most of the algebra in this course will deal with polynomials, it is important that you know what a polynomial is <u>not.</u>

$\frac{4}{x^2} + 3x$ is not a polynomial because $\frac{4}{x^2}$ is the same as $4x^{-2}$. Negative exponents are not allowed on the variables in polynomials.

$\frac{x^2}{4} + 3x$ is a polynomial since the denominator is not a variable.

$\frac{3x - 1}{x + 2}$ is not a polynomial.

EXERCISE 2 1. From the list of expressions below, find an example of each of the following:

(a) monomial _____

(b) constant _____

(c) non-polynomial _____

(d) trinomial _____

$3x^2 - 4x$ $\qquad$ $\frac{-5a^2}{4}$

65 $\qquad$ $w^2 - 4w + 6$

$2x - \frac{7}{x}$ $\qquad$ $3a^2 - \frac{5a}{2}$

2. For each of the following polynomials, list in order the literal coefficients, then the numerical coefficients.

(a) $-3x^2 + 4x - 6$

Literal x^2, x

Numerical $-3, +4, -6$

(b) $7x^2y + 8xy^2$

Literal _____

Numerical _____

(c) $4a^2b - 3ab^2 + 7$

Literal _____

Numerical _____

LIKE AND UNLIKE TERMS

> **Terms which have the same literal coefficients are like terms.**

Remember that the literal coefficient of a term includes the letters and the exponents on the letters.

Example 1 Are $2ab$ and $\frac{-86ab}{3}$ like terms?

Literal coefficient of $2ab$ is ab.

Literal coefficient of $\frac{-86ab}{3}$ is ab.

$\therefore$ $2ab$ and $\frac{-86ab}{3}$ are like terms.

Example 2 Are $3a^2b$ and $3ab$ like terms?

Literal coefficient of $3a^2b$ is a^2b.

Literal coefficient of $3ab$ is ab.

a^2b is not the same as ab.

$\therefore$ $3a^2b$ and $3ab$ are unlike terms.

Reference Read column 1 of page 209 in *Holt Mathematics 3.*

SELF-CORRECTING EXERCISE B

Identify the like terms: One is done for you.

1. x, 2y, 2x, -y, -4x, -x, 3y, 0.5x, 16y

Terms with literal coefficient x:

x, 2x, -4x, -x, 0.5x

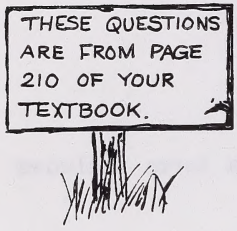
Terms with literal coefficient y:

3y, -y, 2y, 24y

SELF-CORRECTING EXERCISE B (continued)

2. $2b^2$, $3a$, $-3b^2$, $4a$, $-b^2$, $7a$, $10a^2$

Terms with literal coefficient b^2:



Terms with literal coefficient a:

Terms with literal coefficient a^2:

3. $3xy$, $0.4x^2$, $-2y^2$, $-6xy$, $-x^2$, $3y^2$, $-1.5xy$

Literal Coefficient	xy	x^2	y^2
Terms			

4. 3, $-4m$, $-4m^2$, -3, $-3m^3$, $4m$, $-5m^2$, 4, $-m^3$

Literal Coefficient	m	m^2	m^3	none
Terms				

5. $5a^2b$, $-ab$, $-ab^2$, $-2ab^2$, $4ab$, $2a^2b$, $2ab$

Literal Coefficient	a^2b	ab	ab^2
Terms			

Check your answers with those on page 22 of this lesson.
Correct any errors in your work.

EXERCISE 3 Separate the following into groups of like terms.

1. $5x^2$, $4x$, $-2x$, $3x^2$, $-2x^2$, $3x$

_____ are like terms

_____ are like terms

2. 6, $7mn$, $-8m^2n$, $\frac{1}{3}mn$, 9, $-4m^2n$

_____ are like terms

_____ are like terms

_____ are like terms

3. a^2b, $-2ab^2$, $\frac{2}{3}ab$, $6ab$, $-3a^2b$, $-2b^2a$, $6ba$

(Hint: ab is the same as ba (Commutative Law of Multiplication).

_____ are like terms

_____ are like terms

_____ are like terms

4. $6xy$, $6x^2y$, $7yx$, $7yx^2$

_____ are like terms

_____ are like terms

Many mathematicians have formed the habit of always putting the variables in a term in alphabetic order. For example: 7xy instead of 7yx. That way like terms all look the same.

ADDING AND SUBTRACTING LIKE TERMS

Adding and subtracting like terms is easy if you think of the literal coefficients as objects. For example, in the question:

$$3x + 4x$$

you are adding three x's and four x's.

Compare this to adding three Porsches and four Porsches.

In both cases, you ignore the name of the object and add the numbers. Then, you put the object name back in your answer.

$$3 \text{ Porsches} + 4 \text{ Porsches} = 7 \text{ Porsches}$$

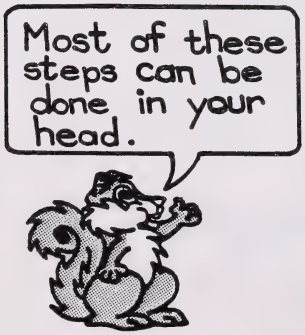
$$3x + 4x = 7x$$

This, of course, is a very simple example. But the same method works for both adding and subtracting like terms.

Example 1 Add $2x$, $3x$ and $-x$.

$$\begin{aligned} & 2x + 3x + (-x) \\ = \ & 2x + 3x - x \\ = \ & (2 + 3 - 1)x \qquad \textit{Remember: } -x = -1x. \\ = \ & 4x \end{aligned}$$

Example 2 Subtract $-3x^2y$ from $-7x^2y$.



$$\begin{aligned} & -7x^2y - (-3x^2y) \\ = \ & -7x^2y + (+3x^2y) \qquad \textit{Change operation to addition.} \\ = \ & (-7 + 3)x^2y \\ = \ & -4x^2y \end{aligned}$$

Example 3 Add 3xy and -7xa.

$$3xy + (-7xa)$$
$$= 3xy - 7xa$$

Since 3xy and -7xa are unlike terms they cannot be added. The final answer is 3xy - 7xa.

ONLY LIKE TERMS CAN BE ADDED OR SUBTRACTED.

SELF-CORRECTING EXERCISE C

Simplify:

1. $3x-(-4x)$

2. $-3x--2x$

3. a^2b-a^2b

4. x^2+x^2

5. $4y^2-5y^2$

6. $-4a^2b-3a^2b$

Check your answers with those on page 23 of this lesson. Correct any errors in your work.

EXERCISE 4

1. Add $-12p$, $-6p$ and $-7p$.

2. Subtract $-3a$ from $5a$.

3. Add $3a^2b$, $-5a^2b$ and $+2a^2b$.

4. Add $8a^3b$, $-9a^3b$ and $-10a^3b$.

5. Subtract $6x^2y$ from $-x^2y$.

6. Subtract $-5x$ from $6x$, then add $-7x$ to the result.

ADDING POLYNOMIALS

Now that you can add terms, you should have no trouble adding polynomials. After all, polynomials are made up of terms.

Rules

1. Only like terms can be added.
2. To add like terms, add their numerical coefficients.

Example 1 Add $3x^2 - x + 4$ and $-x^2 + 4x - 1$.

$$(3x^2 - x + 4) + (-x^2 + 4x - 1)$$

$$= 3x^2 - x + 4 - x^2 + 4x - 1$$

Collect like terms.

$$= 3x^2 - x^2 - x + 4x + 4 - 1$$

$$= (3 + -1)x^2 + (-1 + 4)x + 4 - 1$$

Add like terms.

$$= 2x^2 + 3x + 3$$

Addition of polynomials follows three basic steps.

1. Remove the brackets.
2. Collect like terms in groups. Remember that the sign in front of a term stays with that term.
3. Add like terms.

In some special cases, one or more steps may be omitted.

Example 2 Simplify 3a - 4b + 6a + 6b - 7a.

There are no brackets. Begin by collecting like terms.

$$3a + 6a - 7a - 4b + 6b$$
$$(3 + 6 - 7)\,a + (-4 + 6)\,b \qquad \textit{Add like terms.}$$
$$= 2a + 2b$$

Note that "Simplify", in this case, means to add like terms. Certainly, 2a + 2b is simpler than the original expression.

SELF-CORRECTING EXERCISE D

Simplify.

1. $x + x + x + y + y$

2. $2x^2 + x^2 + 3x - x$

3. $2ab + 4 - ab - 7$

SELF-CORRECTING EXERCISE D (continued)

4. $7x^3 + 2x^2 - 5 - x^2 + 4 + 3x^2$

5. $(y^2 - 4y + 5) + (y^2 - 2y + 1)$

6. $(a^2 - 3a + 2) + (2a^2 + 3a - 4)$

7. $(b^3 - 2b^2 + 1) + (b^4 - b^2 + b)$

8. $(-6x^2 + 4x - 2) + (x^2 - 3x + 3)$

Check your answers with those on page 23 of this lesson.
Correct any errors in your work.

EXERCISE 5

1. Add $5a + 2b - c$ and $a + 5b - 3c$.

2. Add $3x^2 - 2y + 3z$ and $-5x^2 + 4y - 2z$.

3. Add $-a^2 + b^2$, $-6a^2 + 2b^2$ and $-2a^2 + 5b^2$.

4. Simplify each of the following expressions.

   (a) $(5a - 2b) + (-2a + 6b) + (a - 8b)$

   (b) $(3mn - 4pq) + (7mn - 2xy + pq) + (4xy + 3pq) - 10mn$

ADDITIVE INVERSE OF A POLYNOMIAL

A polynomial is a sum of terms.

The additive inverse of a term is the term with its sign changed (opposite sign).

For example, $-3x$ is the additive inverse of $+3x$.

Thus, the additive inverse of a polynomial is the sum of the additive inverses of its terms.

For example, the inverse of $2x^2 - x + 1$ is $-2x^2 + x - 1$.

In simple language, to find the additive inverse of a polynomial, change all its signs.

Reference Read Example 2 on page 37 of *Holt Mathematics 3.*

EXERCISE 6 Find each additive inverse.

	Polynomial	Inverse
1.	$-2x + 3y - 7$	
2.	$x^2 + x^5 - 3x$	
3.	$3a^2b - 3ab + 3$	$-3a^2b + 3ab - 3$
4.	$-x + 2x^2 - 4$	
5.	$6w^2 + 7w + 2$	

SUBTRACTING POLYNOMIALS

> To subtract a polynomial, add its inverse.

You already know how to subtract like terms. Polynomials are subtracted in a similar manner.

Example 1 Subtract $-6x$ from $-4x$.

$$\begin{aligned} & -4x - (-6x) \\ &= -4x + (+6x) \\ &= (-4 + 6)\,x \\ &= 2x \end{aligned}$$

Add the inverse.

Example 2 Subtract $-6x^2 - 7x + 4$ from $4x^2 + x - 7$.

$$(4x^2 + x - 7) - (-6x^2 - 7x + 4)$$

Subtracting $-6x^2 - 7x + 4$ is the same as adding the additive inverse or opposite.

$$= (4x^2 + x - 7) + (+6x^2 + 7x - 4)$$

To get the opposite, change every sign inside the brackets.

Change subtraction to addition.

$$\begin{aligned} &= 4x^2 + x - 7 + 6x^2 + 7x - 4 \\ &= 4x^2 + 6x^2 + x + 7x + (-7) + (-4) \\ &= (4 + 6)\,x^2 + (1 + 7)\,x + (-11) \\ &= 10x^2 + 8x - 11 \end{aligned}$$

Example 3 Simplify.

$$(3a^2 - 6a) + (7a - 4a^2) - (-6a + 4)$$

Change subtraction to addition.

$$= (3a^2 - 6a) + (7a - 4a^2) + (+6a - 4)$$

Remove brackets and extra "+" signs.

$$= 3a^2 - 6a + 7a - 4a^2 + 6a - 4$$

Add like terms.

$$= 3a^2 + (-4a^2) + (-6a) + 7a + 6a + (-4)$$

$$= (3 + -4)\,a^2 + (-6 + 7 + 6)\,a + (-4)$$

$$= -a^2 + 7a - 4$$

SELF-CORRECTING EXERCISE E

1. Subtract $5a + 3b$ from $a - 5b$.

2. Subtract $a^2 - 6a + 3$ from $3a^2 + a - 10$.

3. Simplify: $(x - 5y) - (6x + y) + (-2x + 3y)$

Check your answers with those on page 24 of this lesson.
Correct any errors in your work.

EXERCISE 7

1. Subtract $5x^2 - x + 3$ from $2x - x^2 + 10$.

2. Subtract $4a^2 - 8a + 6$ from $-a^2 + 3a + 7$.

3. Simplify: $(6x - 3y) + (-6x + y) - (3x - 5y)$

4. Subtract $5a^2 - 2a + 3$ from the <u>sum</u> of $a^2 - 6a + 3$ and $3a^2 + a - 5$.

KEY TO SELF-CORRECTING EXERCISES IN LESSON 18

SELF-CORRECTING EXERCISE A, pages 3, 4, and 5.

1. Algebraic expression is a general name for almost any combination of letters, numbers and signs.

2. Polynomial is an expression that is formed by adding one or more terms which have variables with whole number exponents.

3. Terms are the parts of an expression that are added together.

4. Factors are numbers or variables which are multiplied together to form a product.

5. Constants are numbers.

6. Variables are letter symbols used to represent numbers.

7. Literal coefficient of a term contains all the variable factors of the term. It includes the exponents on the variables.

8. Numerical coefficient of a term is the number factor of the term. It includes the sign.

9. Monomial is a polynomial with one term.

10. Binomial is a polynomial with two terms.

11. Trinomial is a polynomial with three terms.

SELF-CORRECTING EXERCISE B, page 7 and 8.

2. $2b^2$, $3a$, $-3b^2$, $4a$, $-b^2$, $7a$, $10a^2$

Terms with literal coefficient b^2:

$2b^2$, $-3b^2$, $-b^2$

Terms with literal coefficient a:

$3a$, $4a$, $7a$

Terms with literal coefficient a^2:

$10a^2$

3.

Literal Coefficient	xy	x^2	y^2
Terms	$3xy$ $-6xy$ $-1.5xy$	$0.4x^2$ $-x^2$	$-2y^2$ $3y^2$

4.

Literal Coefficient	m	m^2	m^3	none
Terms	$-4m$ $4m$	$-4m^2$ $-5m^2$	$-3m^3$ $-m^3$	3 -3 4

5.

Literal Coefficient	a^2b	ab	ab^2
Terms	$5a^2b$ $2a^2b$	$-ab$ $4ab$ $2ab$	$-ab^2$ $-2ab^2$

SELF-CORRECTING EXERCISE C, Page 11

1. $3x - (-4x) = 3x + +4x$
$= 7x$

2. $-3x - -2x = -3x + +2x$
$= -x$ or $-1x$

3. $a^2b - a^2b = 0$

4. $x^2 + x^2 = 2x^2$

5. $4y^2 - 5y^2 = -1y^2$ or $-y^2$

6. $-4a^2b - 3a^2b = -4a^2b + -3a^2b$
$= -7a^2b$

SELF-CORRECTING EXERCISE D, Pages 14 and 15

1. $x + x + x + y + y = 3x + 2y$

2. $2x^2 + x^2 + 3x - x = 3x^2 + 2x$

3. $2ab + 4 - ab - 7 = 2ab - ab + 4 - 7$
$= ab - 3$

4. $7x^3 + 2x^2 - 5 - x^2 + 4 + 3x^2 = 7x^3 + 2x^2 - x^2 + 3x^2 - 5 + 4$
$= 7x^3 + 4x^2 - 1$

5. $(y^2 - 4y + 5) + (y^2 - 2y + 1) = y^2 + y^2 - 4y - 2y + 5 + 1$
$= 2y^2 - 6y + 6$

6. $(a^2 - 3a + 2) + (2a^2 + 3a - 4) = a^2 + 2a^2 - 3a + 3a + 2 - 4$
$= 3a^2 - 2$

7. $(b^3 - 2b^2 + 1) + (b^4 - b^2 + b) = b^4 + b^3 - 2b^2 - b^2 + b + 1$
$= b^4 + b^3 - 3b^2 + b + 1$

8. $(-6x^2 + 4x - 2) + (x^2 - 3x + 3) = -6x^2 + x^2 + 4x - 3x - 2 + 3$
$= -5x^2 + x + 1$

SELF-CORRECTING EXERCISE E, page 19.

1. $(a - 5b) - (5a + 3b)$

$= (a - 5b) + (-5a - 3b)$

$= a - 5b - 5a - 3b$

$= -4a - 8b$

2. $(3a^2 + a - 10) - (a^2 - 6a + 3)$

$= (3a^2 + a - 10) + (-a^2 + 6a - 3)$

$= 3a^2 + a - 10 - a^2 + 6a - 3$

$= 2a^2 + 7a - 13$

3. $(x - 5y) - (6x + y) + (-2x + 3y)$

$= (x - 5y) + (-6x - y) + (-2x + 3y)$

$= x - 5y - 6x - y - 2x + 3y$

$= -7x - 3y$

END OF LESSON 18

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(4) The Lesson Record Form is filled out and the correct lesson label is attached.
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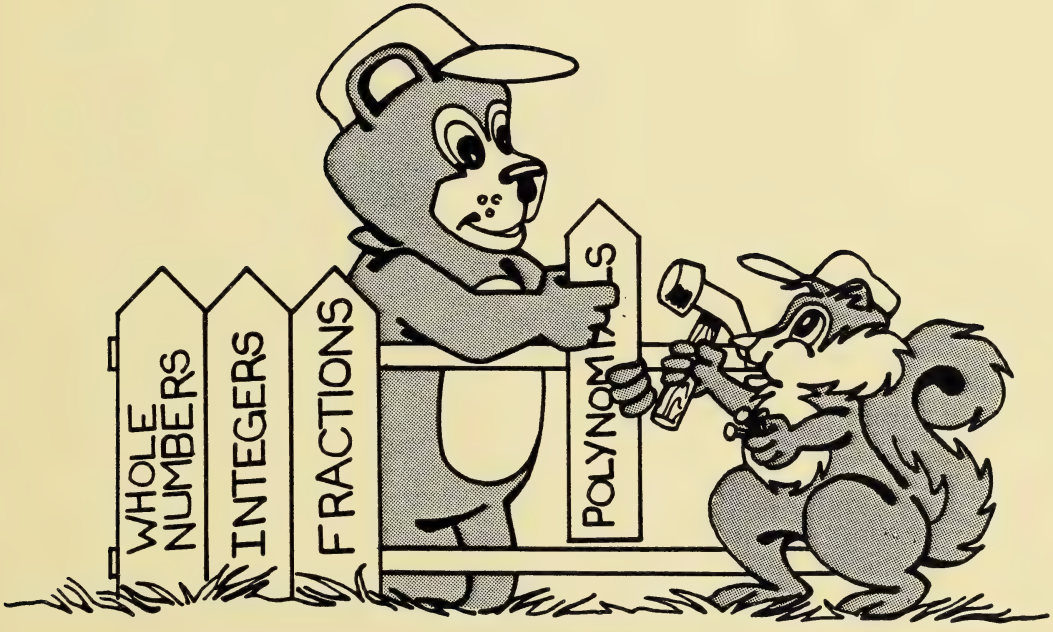
MULTIPLICATION AND DIVISION OF MONOMIALS



In this lesson you will learn about:

- multiplying and dividing monomials
- multiplying and dividing polynomials by monomials
- finding and removing the Greatest Common Factor of a polynomial

KEEPING SKILLFUL

EXERCISE 1 1. Express 420 as a product of its prime factors.

2. Find the Greatest Common Factor of:

(a) 240, 300 and 390

(b) 72, 90 and 126

MULTIPLICATION AND DIVISION USING EXPONENTS

In Lesson 2 you learned to multiply and divide powers. These rules were used:

Exponent Rules

1. To multiply powers with the same base, add the exponents.
2. To divide powers with the same base, subtract the exponents.

The important phrase in both these rules is with the **same base**. You should always be sure your powers have the same base before you use the exponent rules.

Reference

Read pages 55-56 of *Holt Mathematics 3*. Note particularly Example 7 on page 56.

Examples

1. $a^{10} \times a^2 \times a^5$ — *Each power has the same base.*

   $= a^{10+2+5}$ — *Add the exponents.*

   $= a^{17}$

2. $y^8 \div y^3$ — *Each power has the same base.*

   $= y^{8-3}$ — *Subtract the exponents.*

   $= y^5$

3. $x^5 \div x^5$ — *Each power has the same base.*

   $= x^{5-5}$ — *Subtract the exponents.*

   $= x^0$

   $= 1$

When powers have different bases, such as $3^4 \times 2^3$ for example, each power must be evaluated before multiplying.

SELF-CORRECTING EXERCISE A

Simplify:

1. (a) $(3^2)(3^3)(3^2)$ (b) $(x^2)(x^3)(x^4)$

(c) $(m)(m^2)(m^3)$ (d) $(m^0)(m)$

2. (a) $4^5 \div 4^3$ (b) $6^3 \div 6^2$

(c) $y^4 \div y^6 \times y^2$ (d) $a^4 \times a^3 \div a$

(e) $t^3 \div t^9 \times t^8$ (f) $3^7 \div 3^9 \times 3^4$

Check your answers with those on page 22 of this lesson.
Correct any errors in your work.

EXERCISE 2 Perform each of the following operations.

1. $(x^2)(x^5) =$

2. $y^8 \div y^8 =$

3. $a^8 \times a^4 \times a^0 =$

4. $b^7 \div b^0 =$

5. $\frac{(m^2)(m^4)(m^5)}{(m^3)(m^7)} = \frac{m^{2+4+5}}{m^{3+7}} = \frac{m^{11}}{m^{10}} = m^{11-10} = m^1 = m$

6. $(t^0 \times t^4 \times t^5) \div t^9 =$

7. $\frac{(x^5)(x^4)(x^7)}{(x^2)(x^6)} =$

MULTIPLYING MONOMIALS

Monomial

A monomial is a number or the product of a number and powers of one or more variables. For example:

$$-\frac{1}{3}x^3y^2 \qquad 5m^5 \qquad 87a^4bc^2$$

To multiply monomials you should multiply each of the parts separately. Do not combine the powers of different variables.

Reference

Study Example 4 on page 55 of *Holt Mathematics 3.*

Examples

1. $-4x^3y^2 \times 2x^4y^6$

$= (-4 \times 2)(x^3 \times x^4)(y^2 \times y^6)$ *Combine powers with the same base.*

$= -8x^7y^8$

2. $3ab^3c^2 \times a^2b^4$

$= (3 \times 1)(a^1 \times a^2)(b^3 \times b^4)(c^2)$ $a^2b^4 = 1a^2b^4$

$= 3a^3b^7c^2$ $a = a^1$

3. $(-6m^2np^3)(2mn^2)(-mn^3p^2)$

$= (-6 \times 2 \times -1)(m^2 \times m^1 \times m^1)(n^1 \times n^2 \times n^3)(p^3 \times p^2)$

$= 12m^4n^6p^5$

When you get very confident, you may do the middle step in your head.

EXERCISE 3 Multiply.

1. $(-2x^2y)(6xy^2)$ =

2. $(u^2)(-2uv)(5u^2vt)$ =

3. $(-3w^3)(-5tw)(4wt)$ =

4. $(2m^3n^4)(2m^3n^4)$ =

5. $(4x^3y^3)(10xyz^2)$ =

MULTIPLICATION OF A POLYNOMIAL BY A MONOMIAL

> To multiply a polynomial by a monomial, multiply each of the terms of the polynomial by the monomial.

Reference Read page 215 of *Holt Mathematics 3.*

Examples 1. Multiply $6a^2$ and $5a^3 - 4a^2 + 3a - 5$.

$$6a^2(5a^3 - 4a^2 + 3a - 5)$$
$$= (6a^2)(5a^3) + (6a^2)(-4a^2) + (6a^2)(3a) + (6a^2)(-5)$$
$$= 30a^5 - 24a^4 + 18a^3 - 30a^2$$

2. Simplify $(-1)(b^3 - 3b^2 + 5)$.

$$(-1)(b^3 - 3b^2 + 5)$$
$$= (-1)(b^3) + (-1)(-3b^2) + (-1)(5)$$
$$= -b^3 + 3b^2 - 5$$

Note that multiplying by -1 changes only the signs.

SELF-CORRECTING EXERCISE B

Multiply:

1. $2x^2(x^2 - 3x - 4)$

SELF-CORRECTING EXERCISE B (continued)

2. $3y(2y^2 - y + 2)$

3. $(-2x^2 + 4x - 1)(3x)$

4. $(4y^2 - 5y)(-2y)$

5. $4(b^3 - 2b + 3)$

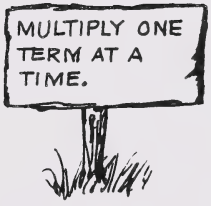
6. $-2a(a^2 - 4)$

Check your answers with those on page 22 of this lesson. Correct any errors.

EXERCISE 4 Multiply.

1. $3x^2(x^2 - x - 7) =$

2. $-a(5a^2 - 1) =$



3. $-xy(x^2 - y^2) =$

4. $3mn(2m^2 - 3m^2n) =$

5. $(a^2 - a - 1)a =$

6. $rs(r^2 - rs + s^2) =$

DIVIDING MONOMIALS

> To divide monomials, divide like factors.

Reference Read column one on page 218 in *Holt Mathematics 3.* Take particular note of Example 3.

Example Simplify $\frac{12xyz^3}{-3z}$.

Method 1

$$\frac{12xyz^3}{-3z}$$

$$= \left(\frac{12}{-3}\right)(x)(y)\left(\frac{z^3}{z}\right) \longleftarrow \frac{z^3}{z} = z^{3-1} = z^2$$

$$= -4xyz^2$$

With practice you should be able to leave out the middle step.

Method 2

$$(12xyz^3) \div (-3z)$$

$$= (12 \div -3)(x)(y)(z^3 \div z)$$

$$= -4xyz^2$$

Note: $\frac{12xyz^3}{-3z}$ and $(12xyz^3) \div (-3z)$ are, of course, the same question.

EXERCISE 5 Divide.

1. $\frac{-25a^2b^2c}{-5abc} =$

2. $\frac{8mn^2}{-2mn} =$

3. $-8a^4 \div -8a^4 =$

4. $\frac{a^2b^2c^2}{abc} =$

5. $21a^4 \div -a^3 =$

DIVISION OF POLYNOMIALS BY MONOMIALS

> To divide a polynomial by a monomial, divide each term separately by the monomial.

Reference Read column 1 on page 220 of *Holt Mathematics 3.*

Examples 1. $\frac{42 - 6a^2}{6} = \frac{42}{6} - \frac{6a^2}{6} = 7 - a^2$

2. $\frac{16xyz^2 - 12xy}{-8x} = \frac{16xyz^2}{-8x} - \frac{12xy}{-8x}$

$= -2yz^2 + \frac{3}{2}y$

The fraction, $\frac{-12}{-8}$ has been reduced to lowest terms.

SELF-CORRECTING EXERCISE C

Divide:

1. $\frac{6x^2 - 12x}{3x}$

2. $\frac{4y^4 - 6y^2 + 2y}{2y}$

3. $\frac{16x^2y + 8xy - 4x}{4x}$

4. $\frac{10x^3 - 15x^2}{5x^2}$

Check your answers with those on page 23 of this lesson. Correct any errors in your work.

EXERCISE 6 Divide.

1. $\frac{3xy - 9x^2y}{-3y} =$

2. $\frac{24a^2bc + 18bc^3}{4c} =$

3. $\frac{m^2q^2r^3 - 4qr^4}{qr^3} =$

4. $\frac{9x^2 - 3x}{3x} =$

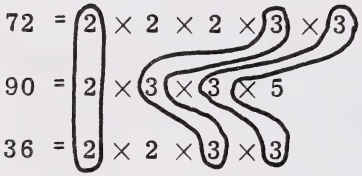
GREATEST COMMON FACTOR

In Lesson 3 you used Greatest Common Factors to reduce fractions. They can also be used in polynomials.

> The Greatest Common Factor, of the terms of a polynomial, is the largest monomial which will divide evenly into every term of the polynomial.

Here again is the GCF for whole numbers. Study the example below.

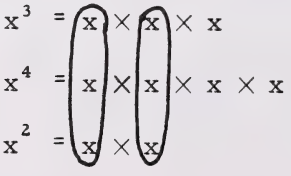
Example Find the GCF of 72, 90, 36.



GCF = $2 \times 3 \times 3 = 18$

The same method is used for monomials.

Example 1 Find the GCF of x^3, x^4 and x^2.



GCF = $x \times x = x^2$

Example 2 Find the GCF of $9a^2b^2$, $6ab^2$, and $-18a^2b$.

$9a^2b^2 = 3 \times 3 \times a \times a \times b \times b$

$6ab^2 = 3 \times 2 \times a \times b \times b$

$-18a^2b = -3 \times 3 \times 2 \times a \times a \times b$

Note that the negative sign is ignored.

GCF $= 3 \times a \times b = 3ab$

SELF-CORRECTING EXERCISE D

Find the GCF for the following:

1. $6x^2, 12x^3, 15x^5$

2. $3ax, 6bx, 9cx$

3. $-9a^2b, 18ab^2, 21a^2b$

4. $6x^2y, 4xy^2, -10x^2y^2$

Check your answers with those on page 23 of this lesson. Correct any errors in your work.

EXERCISE 7 Find the GCF of each of the following sets of terms. Show your work.

1. $4x^2$, $12x$

2. ab, ac, a^2

3. $4ax^2$, $2ax$, $6a^2x$

4. a^2x^3, ax^2, a^3x^2

FACTORING A POLYNOMIAL WHICH HAS A COMMON FACTOR

Earlier in this lesson you learned to multiply a polynomial by a monomial. For example:

$$2x(x + y + z) = 2x(x) + 2x(y) + 2x(z)$$
$$= 2x^2 + 2xy + 2xz$$

Now you will learn to do the inverse operation. You will factor a polynomial into a monomial times a simpler polynomial. For example:

$$2x^2 + 2xy + 2xz = 2x \times (\underline{\quad ? \quad})$$

If you take the common factor, 2x, out of the polynomial, what is the remaining factor?

Consider this simpler case.

$$95 = 5 \times \underline{\quad ? \quad}$$

$$95 = 5 \times \frac{95}{5}$$

You find the unknown factor by dividing 95 by 5.

$$95 = 5 \times 19$$

The same process is used with the polynomial.

$$2x^2 + 2xy + 2xz = 2x\left[\frac{2x^2 + 2xy + 2xz}{2x}\right]$$

$$= 2x\left[\frac{2x^2}{2x} + \frac{2xy}{2x} + \frac{2xz}{2x}\right]$$

$$= 2x(x + y + z)$$

At the beginning of this section, $2x(x + y + z)$ was multiplied to give $2x^2 + 2xy + 2xz$. By factoring, it has now been changed back to $2x(x + y + z)$.

Steps in Common Factoring

1. Find the Greatest Common Factor of the terms of the polynomial.
2. Divide the GCF into each term of the polynomial.

Sometimes the GCF can be found by inspection. At other times a formal method like that on pages 14 and 15 of this lesson must be used.

Example 1 Factor $4a - 4b$.

The common factor is obviously 4.

$$4a - 4b = 4\left[\frac{\cancel{4}a}{\cancel{4}} - \frac{\cancel{4}b}{\cancel{4}}\right]$$

$$= 4(a - b)$$

Check: $4(a - b) = 4a - 4b$

Before you leave a common factoring question be sure that the new polynomial is in simplest form. Check that the terms have no factor in common.

Example 2 Factor $xa + xb$.

The common factor x is obvious.

$$xa + xb = x\left[\frac{\cancel{x}a}{\cancel{x}} + \frac{\cancel{x}b}{\cancel{x}}\right]$$

$$= x(a + b)$$

Check: $x(a + b) = xa + xb$

Example 3 Factor $3xy - 12x^3 + 18x^2y^5$.

The GCF is not obvious here. So, you must first calculate the GCF.

$$3xy = 3 \times x \times y$$
$$12x^3 = 2 \times 2 \times 3 \times x \times x \times x$$
$$18x^2y^5 = 2 \times 3 \times 3 \times x \times x \times y \times y \times y \times y \times y$$

GCF $= 3 \times x = 3x$

$$3xy - 12x^3 + 18x^2y^5$$
$$= 3x\left[\frac{3xy}{3x} - \frac{12x^3}{3x} + \frac{18x^2y^5}{3x}\right]$$
$$= 3x(y - 4x^2 + 6xy^5)$$

Check: $3x(y - 4x^2 + 6xy^5) = 3xy - 12x^3 + 18x^2y^5$

SELF-CORRECTING EXERCISE E

Factor:

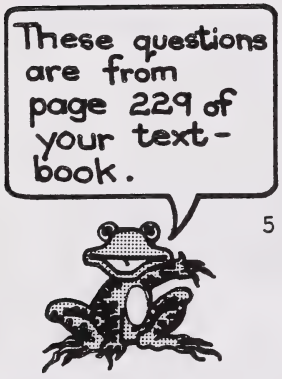
1. $6x^2 + 12x$

2. $5m - 10$

3. $4b^2 - 8b + 4$

SELF-CORRECTING EXERCISE E (continued)

4. $3y^4 - 6y^3 + 12y^2$



5. $2a^2 - 4ab + 8b^2$

Check your answers with those on page 23 of this lesson. Correct any errors in your work.

EXERCISE 8 Factor each of the following polynomials. Check your answer by multiplying.

1. $ax - ay + az$

Check:

2. $14x^2 - 21x$

Check:

EXERCISE 8 (continued)

3. $12m^2 + 18mn$

Check:

4. $m^3n^2 - 12mn$

Check:

5. $4p^3 + 8p$

Check:

6. $a^3 - 7ab^2 + 14ab - a$

Check:

KEY TO SELF-CORRECTING EXERCISES IN LESSON 19

SELF-CORRECTING EXERCISE A, Page 4

1. (a) $(3^2)(3^3)(3^2) = 3^{2+3+2}$
$= 3^7$

(b) $(x^2)(x^3)(x^4) = x^{2+3+4}$
$= x^9$

(c) $(m)(m^2)(m^3) = m^{1+2+3}$
$= m^6$

(d) $(m^0)(m) = m^{0+1}$
$= m^1$
$= m$

2. (a) $4^5 \div 4^3 = 4^{5-3}$
$= 4^2$

(b) $6^3 \div 6^2 = 6^{3-2}$
$= 6^1$
$= 6$

3. (a) $y^4 \div y^6 \times y^2 = y^{4-6+2}$
$= y^0$
$= 1$

(b) $a^4 \times a^3 \div a = a^{4+3-1}$
$= a^6$

(c) $t^3 \div t^9 \times t^8 = t^{3-9+8}$
$= t^2$

(d) $3^7 \div 3^9 \times 3^4 = 3^{7-9+4}$
$= 3^2$

SELF-CORRECTING EXERCISE B, Page 8 and 9

1. $2x^2(x^2 - 3x - 4) = 2x^4 - 6x^3 - 8x^2$
2. $3y(2y^2 - y + 2) = 6y^3 - 3y^2 + 6y$
3. $(-2x^2 + 4x - 1)(3x) = 3x(-2x^2 + 4x - 1)$
$= -6x^3 + 12x^2 - 3x$
4. $(4y^2 - 5y)(-2y) = -8y^3 + 10y^2$
5. $4(b^3 - 2b + 3) = 4b^3 - 8b + 12$
6. $-2a(a^2 - 4) = -2a^3 + 8a$

SELF-CORRECTING EXERCISE C, Page 13.

1. $\frac{6x^2-12x}{3x} = \frac{6x^2}{3x} - \frac{12x}{3x}$

$= 2x - 4$

2. $\frac{4y^4-6y^2+2y}{2y} = \frac{4y^4}{2y} - \frac{6y^2}{2y} + \frac{2y}{2y}$

$= 2y^3 - 3y + 1$

3. $\frac{16x^2y+8xy-4x}{4x} = \frac{16x^2y}{4x} + \frac{8xy}{4x} - \frac{4x}{4x}$

$= 4xy + 2y - 1$

4. $\frac{10x^3-15x^2}{5x^2} = \frac{10x^3}{5x^2} - \frac{15x^2}{5x^2}$

$= 2x - 3$

SELF-CORRECTING EXERCISE D, Page 15.

1. $6x^2 = 2 \times 3 \times x \times x$

$12x^3 = 2 \times 2 \times 3 \times x \times x \times x$

G.C.F. $= 2 \times 3 \times x \times x$

$= 6x^2$

2. $3ax = 3 \times a \times x$

$6bx = 2 \times 3 \times b \times x$

$9cx = 3 \times 3 \times c \times x$

G.C.F. $= 3x$

3. $-9a^2b = -1 \times 3 \times 3 \times a \times a \times b$

$18ab^2 = 2 \times 3 \times 3 \times a \times b \times b$

$21a^2b = 3 \times 7 \times a \times a \times b$

G.C.F. $= 3ab$

4. $6x^2y = 2 \times 3 \times x \times x \times y$

$4xy^2 = 2 \times 2 \times x \times y \times y$

$-10x^2y^2 = -1 \times 2 \times 5 \times x \times x \times y \times y$

G.C.F. $= 2xy$

SELF-CORRECTING EXERCISE E, Page 19 and 20.

1. $6x^2 + 12x = 6x(x + 2)$
2. $5m - 10 = 5(m - 2)$
3. $4b^2 - 8b + 4 = 4(b^2 - 2b + 1)$
4. $3y^4 - 6y^3 + 12y^2 = 3y^2(y^2 - 2y + 4)$
5. $2a^2 - 4ab + 8b^2 = 2(a^2 - 2ab + 4b^2)$

END OF LESSON 19

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REVIEW

This lesson will review the concepts taught in Lessons 1 to 19.

PART A

Lesson 6

1. Perform the indicated operations. (6 marks)

(a) $6.02 \div 10^3 =$ _____

(b) $9 \times 10^4 =$ _____

(c) $0.086 \div 10^2 =$ _____

(d) $28.93 \times 10^6 =$ _____

(e) $0.5 \div 10^{-5} =$ _____

(f) $34 \times 10^{-3} =$ _____

Lesson 6

2. Write the following in scientific notation. (5 marks)

(a) 3097 _____

(b) 56 _____

(c) 321.3 _____

(d) 0.000 55 _____

(e) 0.03 _____

Lesson 6

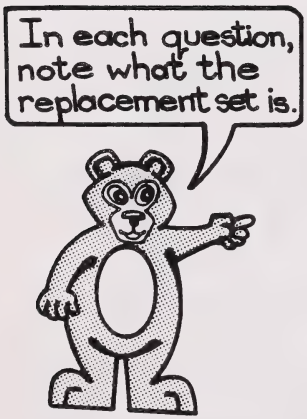
3. Write the following decimals in the form of a fraction. Show your work. (6 marks)

(a) $5.\dot{2}\dot{8}$ (b) $8.2\dot{5}$

Lesson 11

4. Graph the following sets of numbers. (6 marks)

(a) $\{x < 9,\ x \in W\}$



(b) $\{-2 \leq x < 6,\ x \in I\}$

(c) $\{-5 < x \leq 1,\ x \in R\}$

Lesson 11

5. Evalute the following. (4 marks)

(a) $\sqrt{49}$ = _____ (b) $-\sqrt{121}$ = _____

(c) $\sqrt{x^2}$ = _____ (d) $\sqrt{81a^2}$ = _____

6. By comparing the x and y values, write an equation for each of these tables of values. (4 marks)

(a)

x	3	4	5	6	7	8
y	-4	-3	-2	-1	0	1

Equation: _____

x	-6	-4	-2	0	2	4
y	-2	-1	0	1	2	3

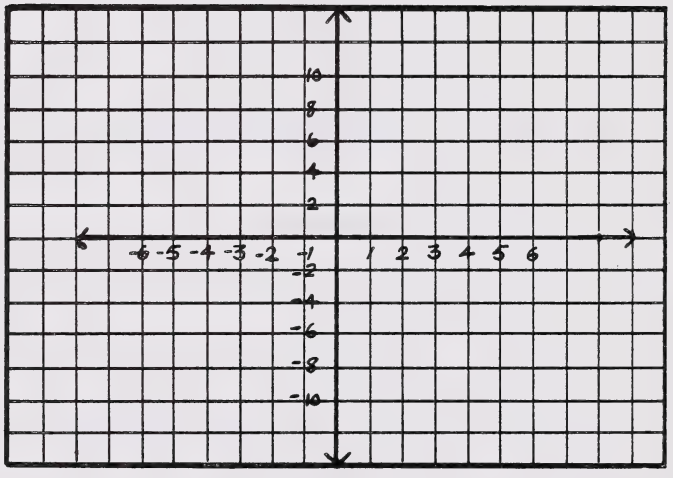
Equation: _____

Lesson 12

7. For each of the following linear equations, fill in a table of values. Then graph the equations. (10 marks)

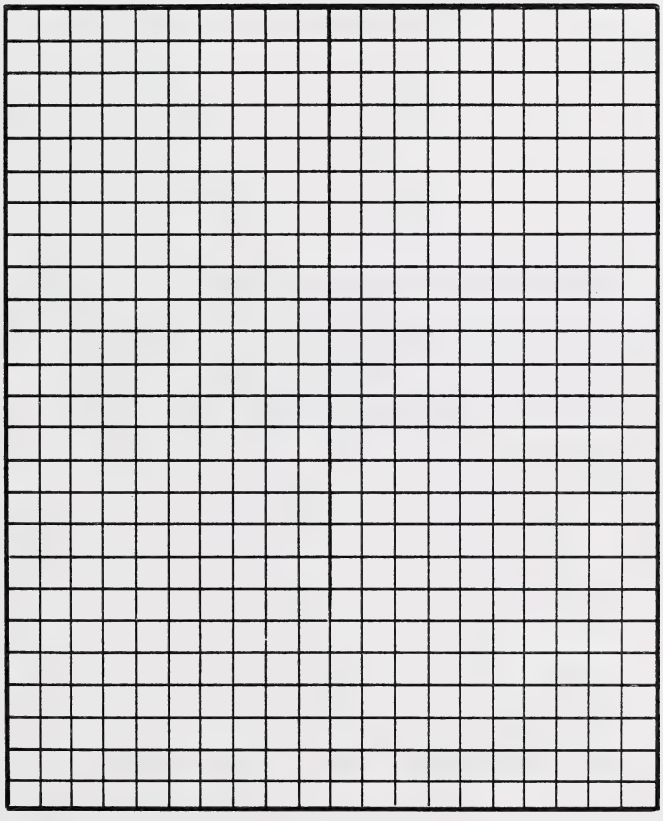
(a) $y = -2x, \ x \in I$

x								
y								



(b) $y = 5 - 2x, x \in R$

x								
y								



Total for Part A _____ (maximum possible: 41 marks)

PART B

Lesson 13

1. Use the given values to evaluate each of the following expressions or formulas. Show your work (6 marks)

(a) $3x^2 - 5x$, for $x = -2$

(b) $P + Prt$, for $P = 21\,000$, $r = .011$, $t = 3$

(c) $\frac{x^2 - x + y}{x - 3y}$, for $x = 3$, $y = 0$

Lesson 13

2. Rearrange the following formulas. Show your work. (4 marks)

(a) $E = IR$, to find I.

(b) $P = R - E$, to find R.

Lesson 14

3. Tickets for the annual Wokatuzzi Spring Concert traditionally sell for \$3.00 for an adult ticket and \$1.00 for a children's ticket. If 200 people attended and the total ticket sales were \$452, how many adults attended? Show all steps. (5 marks)

Lesson 14

4. Ten years from now, Dennis will be just 7 years short of being twice as old as he is now. Find Dennis' present age. Show all steps. (5 marks)

Lesson 15

5. The amount of income tax taken off Joanne's total earnings for the year was $5575.00 if this represents a 25% deduction, what were her total earnings for the year? Show all steps. (5 Marks)

Lesson 17

6. Read each of the following statements. If the statement is true, write TRUE in the space provided. If the statement is false, write FALSE in the space provided. (10 marks)

(a) Every binomial has three terms. _____

(b) The numerical coefficient of x^2 is 0. _____

(c) A polynomial may have 500 terms. _____

(d) The degree of $x - x^2y$ is 3. _____

(e) $2y - y^2 + 9$ is in descending order of degree. _____

(f) Terms are separated by $\times$ or $\div$ signs. _____

(g) A coefficient is a statement that two expressions are equal. _____

(h) $\frac{3}{x} + 2y$ is a polynomial in two variables. _____

(i) The literal coefficient of $-6x^2y^3$ is xy. _____

(j) A constant term has no literal coefficient. _____

Total for Part B _____ (maximum possible: 35 marks)

PART C

Lesson 7

1. For each of the following, put the problem into the general sentence "a is r% of b". Leave a blank for the unknown quantity. (4 Marks)

(a) 520 is what percent of 600?

(b) 420% of 50 is what number?

(c) What is 52 1/2% of 60?

(d) 2% of what number is 58?

Lesson 2

2. Evaluate the following. Show your work. (10 Marks)

(a) 3^4

(b) $(-2)^6$

(c) -2^6

(d) $2(3)^2$

(e) $[2(5)]^2$

(f) $8^5 \div 8^3$

(g) 9^0

(h) $2^5 \times 2^3$

(i) 5^{-3}

(j) $(5^3)^2$

Lesson 18

3. Simplify the following. Show all steps. (10 Marks)

(a) $5a + 3a$

(b) $-3x - (-5x)$

(c) $(6a^2 + 3a + 1) + (a^2 - 2a + 5)$

(d) $(a^2 - 3a - 5) - (a^2 - 4a + 6)$

(e) $5x^2y + 3yx^2 - 6x + 5y^2$

Lesson 19 Simplify the following. Show your work. (10 Marks)

(a) $(3x)(5x^2)$

(b) $(-2x^3)(-3x)$

(c) $(2x)(x^2 + 5x - 3)$

(d) $9x^5 \div 3x$

(e) $\dfrac{18a^2b^3 + 3ab^2 + 15a^3b^2}{3ab^2}$

Total for Part C _____ (maximum possible: 34 marks)

Total for Review _____ (maximum possible: 110 marks)

END OF LESSON 20

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SPECIAL PRODUCTS AND FACTORING

In this lesson you will learn about:

- multiplying polynomials
- taking out common factors
- factoring difference of squares

KEEPING SKILLFUL

Exercise 1

1. Add and subtract

a) $3a + 4a - 7a$

b) $5x^2 - 6x^2 + 2x^2$

c) $3ab + 6 - 2ab - 6ab + 8$

d) $(a^2 - a + 3) + (2a^2 + 5a - 8)$

e) $(7y - 2) + (y^2 - y) + (y - 4) + (y^2 - 3)$

f) $(m^2 + 5m - 3) - (2m^2 - 6m - 1)$

g) $-(3y + 2) + (y - 8) - (y - 3)$

2. Multiply and Divide

a) $(3a)(4b)(-3c)$

b) $(x^2)(2x)(3x^3)$

c) $(2m^5)^3$

d) $a^2(a^2 - 3a + 5)$

e) $mn(m^2n^2 - mn + 1)$

f) $\dfrac{15x^3y}{3x^2}$

g) $\dfrac{39xy^2 + 13xy}{13xy}$

h) $(15b^3 - 25b^2 + 30b) \div 5b$

MULTIPLYING BY A MONOMIAL

In lesson 19 you have multiplied polynomials by monomials. To do so, you must multiply each term of the polynomial by the monomial.

Example 1 $2x^2 (3xy^2 - 4y + 3x^3y)$

$= (2x^2)(3xy^2) + (2x^2)(-4y) + (2x^2)(3x^3y)$

$= (2)(3)(x^2)(x)(y^2) + (2)(-4)(x^2)y + (2)(3)(x^2)(x^3)y$

$= 6x^{2+1}y^2 + -8x^2y + 6x^{2+3}y$

$= 6x^3y^2 - 8x^2y + 6x^5y$

Example 2 $(a^3b - 2ab - 6)(-3ab^2)$

$= (a^3b)(-3ab^2) + (-2ab)(-3ab^2) + (-6)(-3ab^2)$

$= (1)(-3)(a^3)(a)(b)(b^2) + (-2)(-3)(a)(a)(b)(b^2) + (-6)(-3)(a)(b^2)$

$= -3a^{3+1}b^{1+2} + 6a^{1+1}b^{1+2} + 18ab^2$

$= -3a^4b^3 + 6a^2b^3 + 18ab^2$

EXERCISE 2 Multiply.

1. $2a(d - g)$

2. $-3a^2cd^3(a + 4c - 2d)$

3. $(c^2d - cd - 5)(c^2d)$

DIVIDING BY A MONOMIAL

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

Example 1 Divide $-32a^4b^2c^3 + 24a^2bc^2$ by $8abc^2$.

$$= \frac{-32a^4b^2c^3}{8abc^2} + \frac{24a^2bc^2}{8abc^2}$$

$$= \frac{-32}{8}\frac{a^4}{a}\frac{b^2}{b}\frac{c^3}{c^2} + \frac{24}{8}\frac{a^2}{a}\frac{b}{b}\frac{c^2}{c^2}$$

Remember all powers of zero equal 1.

$$= -4a^{4-1}b^{2-1}c^{3-2} + 3a^{2-1}b^{1-1}c^{2-2}$$

$$= -4a^3b^1c^1 + 3a^1b^0c^0$$

$$= -4a^3bc + 3a$$

Example 2 $(10m^2p^4 - 15mp^2) \div (-5mp)$

$$= \frac{10m^2p^4}{-5mp} - \frac{15mp^2}{-5mp}$$

$$= \frac{10}{-5}\frac{m^2}{m}\frac{p^4}{p} + \frac{-15}{-5}\frac{m}{m}\frac{p^2}{p}$$

A negative by a negative results in a positive.

$$= -2m^{2-1}p^{4-1} + 3m^{1-1}p^{2-1}$$

$$= -2m^1p^3 + 3m^0p^1$$

$$= -2mp^3 + 3p$$

Exercise 3 1. Divide $12ax^2 - 15a^2x$ by $3ax$.

2. Divide $32m^3n^4 - 40m^2n^2$ by $8m^2n^2$.

3. $(x^3 + x^2y + xy^2) \div (-x) =$

FINDING A COMMON FACTOR

To find the Greatest Common Factor of a group of terms, start by expressing each term as a product of its prime factors.

Example 1 Find the Greatest Common Factor of the terms of $6am - 30a$.

$6am = 2 \times 3 \times a \times m$

$30a = 2 \times 3 \times 5 \times a$

Circle the common factors.

$GCF = 2 \times 3 \times a = 6a$

The GCF is the product of the common factors.

Example 2 Find the GCF of the terms of $x^2y^2 - 4x^2y - xy^2$.

$x^2y^2 = x \times x \times y \times y$

$-4x^2y = 2 \times 2 \times x \times x \times y$

$-xy^2 = x \times y \times y$

Circle the factors that are common to each of the terms.

$GCF = x \times y = xy$

Negative values are not considered to be part of a common factor.

EXERCISE 4 Find the Greatest Common Factor of the terms of each of the following polynomials. Show your work.

1. $4x^2 - 12x$

2. $3a - 6b + 9$

3. $3x^2 + 15x^3$

4. $abc + abd + abe$

COMMON FACTORING

To factor a polynomial divide the same common factor out of each term in the polynomial. To factor fully, divide by the Greatest Common Factor.

Example 1 Consider $6ax + 12ab$. You could factor (divide) out the "a" which is common and get:

$$a(6x + 12b)$$

However, the expression would not be completely factored since a common factor of 6 remains.

Thus, $6ax + 12ab = 6a(x + 2b)$.
$6a(x + 2b)$ is the complete factorization.

With a relatively simple question like this one you can probably do the division in your head, but remember what the extra steps are:

$$6ax + 12ab = 6a\,\frac{6ax}{6a} + \frac{12ab}{6a} = 6a\,\frac{6}{6}\,\frac{ax}{a} + \frac{12}{6}\,\frac{ab}{a} =$$

$$6a(1a^{1-1}x + 2a^{1-1}b) = 6a(1a^{0}x + 2a^{0}b)$$

$$= 6a(x + 2b)$$

Example 2 Factor $a^2bc - ab^2c + abc$.

$$a^2bc = a \times a \times b \times c$$

$$ab^2c = a \times b \times b \times c$$

$$abc = a \times b \times c$$

$$GCF = a \times b \times c = abc$$

$$a^2bc - ab^2c + abc = abc\ \frac{a^2bc}{abc} - \frac{ab^2c}{abc} + \frac{abc}{abc}$$

$$= abc\ \frac{a^2b\ c}{a\ b\ c} - \frac{a\ b^2c}{a\ b\ c} + \frac{a\ b\ c}{a\ b\ c}$$

$$= abc(a^{2-1}b^{1-1}c^{1-1} - a^{1-1}b^{2-1}c^{1-1} + a^{1-1}b^{1-1}c^{1-1})$$

$$= abc(a^{1}b^{0}c^{0} - a^{0}b^{1}c^{0} + a^{0}b^{0}c^{0})$$

$$= abc(a - b + 1)$$

Always give your answer a last careful look to be sure you have not missed a common factor.

EXERCISE 5 Factor the following polynomials.

1. $5m - 15n$

2. $ax + ay + az$

3. $2p^3 - 4p^2$

4. $5x - 20x^2 + 25x^3$

5. $4bc - 8b^2c^2$

MULTIPLICATION OF BINOMIALS

Reference Read page 224 of *Holt Mathematics 3.*

On page 224, the Distributive Property is used to multiply binomials. Another method using rectangles is also shown.

The purpose of both these methods is to multiply each term in the first binomial by each term in the second binomial. The methods are simply systematic ways of doing this.

Example 1 below illustrates a popular method.

Example 1 $(x - 1)(x + 6)$

$$= x(x + 6) - 1(x + 6)$$

$$* = (x)(x) + (x)(+6) + (-1)(x) + (-1)(+6)$$

$$= x^2 + 6x - x - 6$$

$$= x^2 + 5x - 6$$

Line * is the important line. Take a closer look at it.

$(x-1)(x+6)$ $(x)(x)$ is the product of the first terms in each bracket. **F**

$(x-1)(x+6)$ $(x)(+6)$ is the product of the outside terms. **O**

$(x-1)(x+6)$ $(-1)(x)$ is the product of the inside terms. **I**

$(x-1)(x+6)$ $(-1)(+6)$ is the product of the last terms in each bracket. **L**

This method of keeping track of products is called the FOIL method, for obvious reasons.

Example 2 $(x + 5)(x + 3)$

First	$(x)(x) = x^2$	*This part can be done in your head.*
Outside	$(x)(+3) = 3x$	
Inside	$(+5)(x) = 5x$	
Last	$(+5)(+3) = 15$	

So, $(x + 5)(x + 3)$

$= x^2 + 3x + 5x + 15$

$= x^2 + 8x + 15$

Example 3 $(4a - 5b)(2a - b)$

$= (4a)(2a) + (4a)(-b) + (-5b)(2a) + (-5b)(-b)$

$= 8a^2 - 4ab - 10ab + 5b^2$

$= 8a^2 - 14ab + 5b^2$

Usually two of the four products are like terms. So you end up with a trinomial as the final answer. However, this is not always true as you will see later.

SELF-CORRECTING EXERCISE A

Multiply:

1. $(x + 3)\ (x - 4)$

2. $(b + 1)\ (b + 2)$

3. $(2x - 3)\ (3x + 4)$

4. $(a - b)\ (-3a + b)$

5. $(3x - 2y)\ (x + y)$

Check your answers with those on page 25 of this lesson. Correct any errors in your work.

EXERCISE 6 Multiply.

1. $(a + 5)(a - 7)$

2. $(2m - 4)(m + 6)$

3. $(3c - 4d)(2c - 5d)$

4. $(2 - x)(3 - 5x)$

5. $(2y - 5x)(3y + x)$

MULTIPLYING BINOMIALS MENTALLY

You have probably heard of people who can multiply mentally with great speed. They have learned to multiply using patterns.

If you look for patterns in the multiplying of binomials, you should be able to multiply mentally.

Consider: $(x + 3)(x + 5)$.

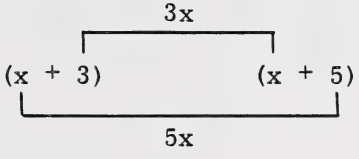
$$= x^2 + 5x + 3x + 15$$
$$= x^2 + 8x + 15$$

Where did each term in the product come from?

x^2 is the product of the first terms in each bracket.

$+15$ is the product of the last terms in each bracket.

$8x$ is the sum of the inner and outer product.



Knowing this, you should be able to multiply simple binomials without writing down intermediate steps.

Example 1 $(x - 2)(x - 4)$

Middle term comes from $(-4x) + (-2x)$.

$$= x^2 - 6x + 8$$

Example 2 $(a + 10)(a - 7)$

Middle term comes from $(-7a) + (+10a)$.

$$= a^2 + 3a - 70$$

Example 3 $(w - 11)(w + 4)$

Middle term comes from (+4w) + (-11w).

$= w^2 - 7w - 44$

EXERCISE 7 Multiply mentally.

1. $(a + 2)(a + 5) =$

2. $(m - 3)(m - 4) =$

3. $(x - 1)(x + 1) =$

4. $(x - 2)(x + 1) =$

5. $(w + 3)(w - 7) =$

SPECIAL PRODUCTS

Some binomials, when multiplied, result in a particular pattern.

Example 1 Multiply: $(x - 6)(x + 6)$.

$$(x - 6)(x + 6) = x^2 + 6x - 6x - 36$$

$$= x^2 - 36$$

Notice that the two middle terms cancelled each other out. That is because their numerical coefficients are the same, except for the sign.

Take a good look at the original binomials. Can you see a pattern that might make you expect the answer? Here is another one to look at.

Example 2 Multiply: $(y + 5)(y - 5)$.

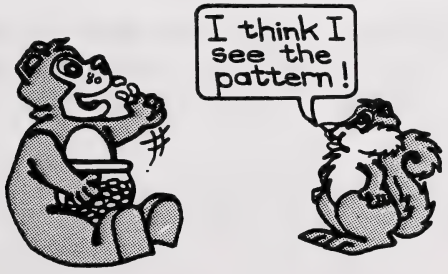
$$(y + 5)(y - 5) = y^2 - 5y + 5y - 25$$

$$= y^2 - 25$$

Difference of Squares

This pattern is called Difference of Squares because the product consists of a perfect square minus another perfect square.

In Exercise 7 some products will be Difference of Squares and some will not. Multiply as you normally would, but also be thinking about why some products are Difference of Squares and why some are not.



EXERCISE 8 Multiply.

1. $(a - 2)(a - 2)$

2. $(a + 2)(a + 2)$

3. $(a + 2)(a - 2)$

4. $(2x - 3)(2x + 3)$

5. $(3x + 5)(3x - 5)$

6. Which of your answers to questions 1 to 5 were Difference of Squares? _____

_____

Did you notice that when the first terms were the same and the last terms were the same, and the signs were opposite, that the product was a Difference of Squares?

FACTORING DIFFERENCE OF SQUARES

Now that you know how to multiply two binomials and get a difference of squares, you should learn how to do the operation backwards (the inverse operation).

Rule

> The difference of the squares of two numbers equals their sum times their difference.

Example 1 Factor $b^2 - 36$.

$b^2 - 36$ is the same as $b^2 - 6^2$. So, it is the difference of the squares of b and 6.

The sum of b and 6 is $(b + 6)$. The difference of b and 6 is $(b - 6)$.

Thus, $b^2 - 36 = b^2 - 6^2 = (b + 6)(b - 6)$.

Check: $(b + 6)(b - 6) = b^2 - 6b + 6b - 36$

The product of the factors should equal the original polynomial. → $= b^2 - 36$

Example 2 Factor $49a^2 - b^2$.

$= (7a)^2 - b^2$

$= (7a + b)(7a - b)$

$49a^2 = 7a \times 7a$

$b^2 = b \times b$

Check: $(7a + b)(7a - b) = 49a^2 - 7ab + 7ab - b^2$

$= 49a^2 - b^2$

As long as you can find the square root of each of the terms and there is a minus sign between them, you have a difference of squares.

Example 3 Factor $1 - 25c^2$.

$= 1^2 - (5c)^2$

$= (1 + 5c)(1 - 5c)$

Remember that $1^2 = 1 \times 1 = 1$.

Check: $(1 + 5c)(1 - 5c) = 1 - 5c + 5c - 25c^2$

$= 1 - 25c^2$

SELF-CORRECTING EXERCISE B

Factor:

1. $y^2 - 1$

2. $x^2 - 4$

3. $9x^2 - 25$

4. $16 - y^2$

5. $64 - 9m^2$

6. $x^2 - y^2$

Check your answers with those on page 25 of this lesson.
Correct any errors in your work.

EXERCISE 9 Factor each expression. Check your answer by multiplying.

1. $m^2 - n^2$ Check:

2. $16 - k^2$ Check:

3. $4a^2 - 1$ Check:

4. $y^2 - 4$ Check:

5. $4x^2 - 25y^2$ Check:

FACTORING COMBINATIONS

You have now learned two factoring methods:

- common factor
- difference of squares

Often both these methods must be used to completely factor a polynomial.

The common factor should always be removed first.

Example 1 Factor $3m^2 - 48$.

At first glance this polynomial seems unfactorable. But, if you remember that $48 = 3 \times 16$ you will realize that 3 is a common factor.

$$3m^2 - 48 = 3m^2 - 3(16)$$
$$= 3(m^2 - 16)$$

Now that the 3 has been factored out, you have a simple difference of squares.

$$m^2 - 16 = m^2 - 4^2$$
$$= (m + 4)(m - 4)$$

Thus, $3m^2 - 48 = 3(m^2 - 16)$
$= 3(m + 4)(m - 4)$

Example 2 Factor $x^4 - 16$

$x^4 = x^2 \times x^2 = (x^2)^2$ $\qquad$ $16 = 4 \times 4 = 4^2$

So, $x^4 - 16 = (x^2)^2 - 4^2$
$= (x^2 + 4)(x^2 - 4)$

However, $x^2 - 4$ is also a difference of squares. → $x^2 - 4 = x^2 - 2^2$
$= (x + 2)(x - 2)$

Thus, $x^4 - 16 = (x^2 + 4)(x^2 - 4)$
$= (x^2 + 4)(x + 2)(x - 2)$

Example 3 Factor $m^5 - 81m$.

m is common to both terms.

$$m^5 - 81m = m(m^4 - 81)$$
$$= m(m^2 + 9)(m^2 - 9)$$
$$= m(m^2 + 9)(m + 3)(m - 3)$$

$m^2 - 9$ is also a difference of squares.

SELF-CORRECTING EXERCISE C

Factor completely. Take out a common factor first.

1. $3y^2 - 75$

2. $5x^4 - 80$

3. $4y^2 - 36$

4. $3x^3 - 75x$

Check your answers with those on page 25 of this lesson.
Correct any errors in your work.

EXERCISE 10 Factor completely. Be sure to take out a common factor first, if possible. Check your answers by multiplying.

1. $ax^2 - 100a$ Check:

2. $2x^2 - 128$ Check:

3. $16m^4 - n^4 =$ Check:

4. $64x^4 - 4y^4 =$ Check:

IRREDUCIBLE POLYNOMIALS

You have already had experience with expressing numbers as a product of prime numbers. For example:

$$36 = 2 \times 2 \times 3 \times 3$$

When you factor a polynomial completely, you are expressing it as a product of prime or irreducible polynomials. Like prime numbers, these polynomials cannot be broken down any further.

Occasionally, you were given prime numbers to factor. Then you would write something like:

$$29 = \text{prime, or, } 29 = 29$$

In the same way you may be given an irreducible polynomial to factor.

Polynomials like $x + 4$, $3x + 1$ and $a + b - c$ are obviously irreducible. However, some are not so obvious.

One such polynomial is the sum of squares, such as:

$$x^2 + 4$$

Many students feel that since $x^2 - 4$ is factorable, $x^2 + 4$ should also be factorable.

However, study the list of possibilities below:

1. $(x + 2)(x - 2) = x^2 - 2x + 2x - 4 = x^2 - 4$
2. $(x + 2)(x + 2) = x^2 + 2x + 2x + 4 = x^2 + 4x + 4$
3. $(x - 2)(x - 2) = x^2 - 2x - 2x + 4 = x^2 - 4x + 4$
4. $(x + 4)(x + 1) = x^2 + 1x + 4x + 4 = x^2 + 5x + 4$
5. $(x - 4)(x - 1) = x^2 - 1x - 4x + 4 = x^2 - 5x + 4$

Since no combination of factors results in $x^2 + 4$ as a product, you must conclude that $x^2 + 4$ cannot be factored.

Exercise 11

1. Make four different "educated guesses" for binomial factors of $x^2 + 9$. Multiply each to show that the product is not $x^2 + 9$. Two are started for you.

(a) $(x + 3)(x + 3) =$

(b) $(x - 9)(x - 1) =$

(c)

(d)

2. Explain why $x^2 + 9$ cannot be factored.

KEY TO SELF-CORRECTING EXERCISES IN LESSON 21

SELF-CORRECTING EXERCISE A, page 11.

1. $(x+3)(x-4) = x^2-4x+3x-12$
$= x^2-x-12$

2. $(b+1)(b+2) = b^2+2b+b+2$
$= b^2+3b+2$

3. $(2x-3)(3x+4) = 6x^2+8x-9x-12$
$= 6x^2-x-12$

4. $(a-b)(-3a+b) = -3a^2+ab+3ab-b^2$
$= -3a^2+4ab-b^2$

5. $(3x-2y)(x+y) = 3x^2+3xy-2xy-2y^2$
$= 3x^2+xy-2y^2$

SELF-CORRECTING EXERCISE B, page 18.

1. $y^2-1=(y-1)(y+1)$

2. $x^2-4=(x-2)(x+2)$

3. $9x^2-25=(3x-5)(3x+5)$

4. $16-y^2=(4-y)(4+y)$

5. $64-9m^2=(8-3m)(8+3m)$

6. $x^2-y^2=(x-y)(x+y)$

SELF-CORRECTING EXERCISE C, page 21.

1. $3y^2-75 = 3\left(y^2-25\right)$
$= 3(y-5)(y+5)$

2. $5x^4-80 = 5\left(x^4-16\right)$
$= 5\left(x^2-4\right)\left(x^2+4\right)$
$= 5(x-2)(x+2)\left(x^2+4\right)$

3. $4y^2-36 = 4\left(y^2-9\right)$
$= 4(y-3)(y+3)$

4. $3x^3-75x = 3x\left(x^2-25\right)$
$= 3x(x-5)(x+5)$

END OF LESSON 21

LESSON RECORD FORM

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1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do not enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

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FACTORING

In this lesson you will learn about:

- factoring perfect squares
- factoring trinomials of the form $x^2 + bx + c$

KEEPING SKILLFUL

EXERCISE 1

1. Add $6x^2 - 5x + 4$ and $-3x^2 + 4x - 7$.

2. Subtract $x^3 - x + 6$ from $2x^3 + x + 6$.

3. Multiply $(2x + 3)$ by $(x - 4)$.

4. Divide $16x^2y - 8x^3y^2 + 4xy^3$ by $-2xy$.

PERFECT SQUARES

A perfect square is a number which can be formed by multiplying the number by itself. For example:

$8 \times 8 = 64$ so, 64 is a perfect square.

$13 \times 13 = 169$ so, 169 is a perfect square.

A perfect square can also be a polynomial which is the product of the polynomial and itself. For example:

$(x + 2)(x + 2) = x^2 + 4x + 4$, so, $x^2 + 4x + 4$ is a perfect square.

$(2x - 3)(2x - 3) = 4x^2 - 12x + 9$, so, $4x^2 - 12x + 9$ is a perfect square.

$(3x^2)(3x^2) = 9x^4$, so, $9x^4$ is a perfect square.

In this lesson you will be looking primarily at perfect squares which are trinomials.

Remember that when you multiply two binomials you usually get a trinomial as a product. So, trinomial perfect squares are squares of binomials.

Example 1 Find the perfect square of $(a + b)^2$.

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b) \\ &= a^2 + ab + ab + b^2 \\ &= a^2 + 2ab + b^2\end{aligned}$$

Note the form of the trinomial in Example 1:

1. The first and last terms are perfect squares, $a \times a$ and $b \times b$.

2. The middle term is twice the product of a and b.

Example 2 $(a - b)^2 = (a - b)(a - b)$

$= a^2 - ab - ab + b^2$

$= a^2 - 2ab + b^2$

Only the sign is different from $(a + b)^2$.

Example 3 $(2x - 3y)^2 = (2x - 3y)(2x - 3y)$

$= 4x^2 - 6xy - 6xy + 9y^2$

$= 4x^2 - 12xy + 9y^2$

1. This first term is $2x \times 2x = 4x^2$.
2. The last term is $-3y \times -3y = +9y^2$.
3. The middle term is $(2 \times -6xy) = (2 \times 2x \times -3y) = -12xy$.

EXERCISE 2 Multiply the following.

1. $(4a - 3)^2 =$

2. $(x + y)^2 =$

3. $(6 - 5d)^2 =$

Look carefully at your answers to Exercise 2. Check that your answers are of the form listed below.

Trinomial Square Form

First Term = square of the first term in the binomial.

Middle Term = twice the product of the two terms in the binomial.

Last Term = square of the last term in the binomial.

Example 1 Use the trinomial square form to find the product: $(x - 3y)^2$.

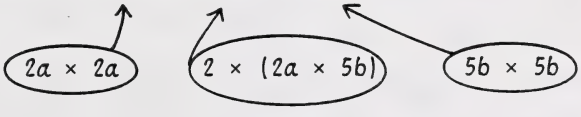
First Term = $x \times x = x^2$

Middle Term = $2(x \times -3y) = -6xy$

Last Term = $-3y \times -3y = +9y^2$

Thus, $(x - 3y)^2 = x^2 - 6xy + 9y^2$.

Example 2 $(2a + 5b)^2 = 4a^2 + 20ab + 25b^2$



EXERCISE 3 Fill in the blanks with the correct term. Do not forget to include the sign.

1. $(x + y)^2 = x^2$ _____ $+ y^2$

2. $(y - 5)^2 =$ _____ $-10y + 25$

3. $(m + 3)^2 = m^2 + 6m$ _____

4. $(5x - 1)^2 =$ _____ $-10x$ _____

5. $(3a - 1)^2 = 9a^2$ _____ _____

6. $(5x - 7y)^2 =$ _____ _____ $+ 49y^2$

FACTORING PERFECT SQUARES

Now that you know how a trinomial perfect square is put together, you ought to be able to take one apart.

Example 1 Factor $a^2 + 6ab + 9b^2$.

1. Is the first term a perfect square? *yes*

   $a^2 = a \times a$

2. Is the last term a perfect square? *yes*

   $9b^2 = 3b \times 3b$

3. Is the middle term twice the product of the two terms in the binomial? *yes*

   $2 \times a \times 3b = 6ab$

Then, $a^2 + 6ab + 9b^2$ must be the square of the binomial $(a + 3b)$.

$$a^2 + 6ab + 9b^2 = (a + 3b)^2$$

Example 2 Factor $100x^2 - 60xy + 9y^2$.

1. Is the first term a perfect square? *yes*

   $100x^2 = 10x \times 10x$

2. Is the last term a perfect square? *yes*

   $9y^2 = 3y \times 3y$

3. Is the middle term twice the product of the two terms in the binomial? *no*

   The middle term is $-60xy$ [$2 \times 10x \times 3y = 60xy$].

How do you get the minus sign? Look back at Examples 2 and 3 on page 4 and Example 1 on page 3. In each case, the middle term in the product was negative and so was the second term in the original binomial.

Note that the middle term is negative for each product below:

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(2x - 3y)^2 = 4x^2 - 12xy + 9y^2$$

$$(x - 3y)^2 = x^2 - 6xy + 9y^2$$

Does $100x^2 - 60xy + 9y^2 = (10x - 3y)^2$?

$$(10x - 3y)^2 = (10x - 3y)(10x - 3y)$$

$$= 100x^2 - 30xy - 30xy + 9y^2$$

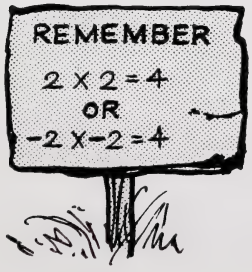
$$= 100x^3 - 60xy + 9y^2$$

So, $100x^2 - 60xy + 9y^2 = (10x - 3y)^2$.

Steps in Factoring Perfect Squares

1. Find the square root of the first term.
2. Find the square root of the last term.
3. Check to be sure that the middle term (except for the sign) is twice the product of the square roots from 1 and 2 above.
4. Form the binomial using these two square roots:
   (a) If the middle term of the trinomial is negative, place minus sign between them.
   (b) If the middle term is positive, place a plus sign between them.

Example 3 Factor $a^2b^2 - 4ab + 4$.



1. $\sqrt{a^2b^2} = ab$, or, $-(ab)$
2. $\sqrt{4} = 2$, or, -2
3. $2 \times ab \times 2 = 4ab$

These steps may be done in your head.

4. $a^2b^2 - 4ab + 4 = (ab - 2)^2$

The same signs.

SELF-CORRECTING EXERCISE A

Factor each of the following trinomials.

1. $a^2 - 10a + 25$

2. $x^2 + 12x + 36$

3. $4y^2 - 20y + 25$

4. $144a^2 + 24a + 1$

Check your answers with those on page 22 of this lesson.
Correct any errors in your work.

EXERCISE 4 Factor each of the following trinomials. Check your answers by multiplying. The first one is done for you.

1. $49 + 14a + a^2 =$

$\sqrt{49} = 7$ or -7

$\sqrt{a^2} = a$ or $-a$

$2 \times a \times 7 = 14a$

So, $49 + 14a + a^2 = (7+a)^2$

Check:

$(7+a)^2 = (7+a)(7+a)$

$= 49 + 7a + 7a + a^2$

$= 49 + 14a + a^2$ ✓

2. $a^2 - 24a + 144 =$ Check:

3. $a^2 - 4a + 4 =$ Check:

4. $9x^2 + 24xy + 16y^2 =$ Check:

TRINOMIALS OF THE FORM $x^2 + bx + c$

When you multiply two binomials, you usually get a trinomial as the product.

If your binomials are of the general form $(x + p)$, where the first numerical coefficient is one, then your answer will be of the form $x^2 + bx + c$.

Example 1 $(x + 3)(x - 2) = x^2 - 2x + 3x - 6$

$= x^2 + x - 6$

Take a close look at the answer.

The first term is the product of the first terms in the binomials.

$$x \times x = x^2$$

The middle term is the sum of the products of the inner and outer terms in the binomials.

$$(x + 3)(x - 2)$$

($+3x$ above, $-2x$ below)

$+3x - 2x = +x$

The last term is the product of the last terms in the binomials.

$$+3 \times -2 = -6$$

Example 2 $(a - 7)(a - 4) = a^2 - 4a - 7a + 28$

$= a^2 - 11a + 28$

The last term is the product of -7 and -4.

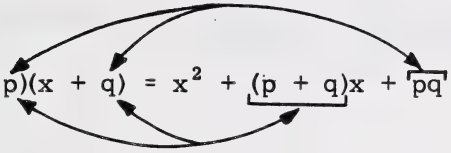
$(a - 7)(a - 4) = a^2 - 11a + 28$

The middle term contains the sum of -7 and -4.

The fact that a is used instead of x makes no difference.

In general,

$$(x + p)(x + q) = x^2 + px + qx + pq$$
$$= x^2 + (p + q)x + pq$$



Product of p and q.

Sum of p and q.

Study the examples below:

When both p and q are positive, the middle term is positive.

(a) $(x + 2)(x + 6) = x^2 + 8x + 12$

(b) $(x + 9)(x + 3) = x^2 + 12x + 27$

(c) $(a - 2)(a - 1) = a^2 - 3a + 2$

When both p and q are negative, the middle term is negative.

(d) $(y - 3)(y - 12) = y^2 - 15y + 36$

When p and q are of the same sign, the last term is positive.

(e) $(x - 3)(x + 4) = x^2 + x - 12$

(f) $(x + 9)(x - 7) = x^2 + 2x - 63$

(g) $(a - 8)(a + 1) = a^2 - 7a - 8$

(h) $(b + 5)(b - 6) = b^2 - b - 30$

When the signs in the binomials are different, the last term in the trinomial is always negative but the sign of the middle term varies.

FACTORING $x^2 + bx + c$

From the observations on pages 10 and 11 you know that if you are going to factor a trinomial of the form $x^2 + bx + c$, you must find two numbers whose sum is b and whose product is c.

For convenience, p and q are often used to represent these numbers. Thus:

$$p + q = b,$$

$$p \times q = c,$$

and, $x^2 + bx + c = (x + p)(x + q)$.

Example 1 Factor $x^2 + 8x + 15$.

$x^2 + 8x + 15 = (x \_\_\_\_\_)(x \_\_\_\_\_)$ *These two numbers must have a sum of 8 and a product of 15.*

$p + q = 8$ *If $p + q = 8$, p and q must both be positive to give a positive sum.*

$p \times q = 15$ *If $p \times q = 15$, then p and q must have the same sign. (Like signs give a positive product.)*

Think of possible factors of 15 and test these to see if their sum is 8.

Possible Factors	$p \times q$	$p + q$	Do they fit?
1, 15	$15 \times 1 = 15$	$15 + 1 = 16$	No
3, 5	$5 \times 3 = 15$	$5 + 3 = 8$	Yes

So, $x^2 + 8x + 15 = (x + 5)(x + 3)$.

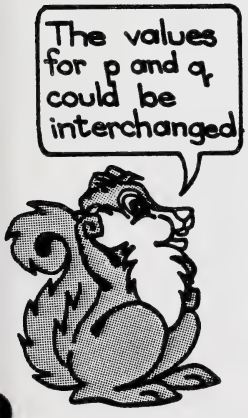
Check: $(x + 5)(x + 3) = x^2 + 3x + 5x + 15$

$= x^2 + 8x + 15$

Example 2 Factor $t^2 - 10t + 24$.

$p \times q = 24$ *If $p \times q = +24$, then p and q must have the same sign.*

$p + q = -10$ *If $p + q = -10$, then p and q must be negative, since two positive numbers will never give a negative sum.*



Possible Factors	$p \times q$	$p + q$	Do they fit?
-24, -1	$-24 \times -1 = +24$	$-24 - 1 = -25$	No
-12, -2	$-12 \times -2 = +24$	$-12 - 2 = -14$	No
-8, -3	$-8 \times -3 = +24$	$-8 - 3 = -11$	No
-6, -4	$-6 \times -4 = +24$	$-6 - 4 = -10$	Yes

Thus, $p = -6$ and $q = -4$. You could also have set $p = -4$ and $q = -6$.

So, $t^2 - 10t + 24 = (t - 6)(t - 4)$.

Check: $(t - 6)(t - 4) = t^2 - 4t - 6t + 24$

$= t^2 - 10t + 24$

Much of the work in these solutions can be done in your head. Your solution might look like Example 3 below.

Example 3 Factor $x^2 - 19x + 48$.

$p + q = -19$

$p \times q = +48$ $\Rightarrow$ $p = -16$ and $q = -3$

$x^2 - 19x + 48 = (x - 16)(x - 3)$

Check: $(x - 16)(x - 3) = x^2 - 3x - 16x + 48$

$= x^2 - 19x + 48$

SELF-CORRECTING EXERCISE B

Factor each of the following trinomials.

1. $a^2 + 5a + 6$

2. $b^2 + 4b + 3$

3. $y^2 - 7y + 12$

4. $x^2 + 9x + 20$

Check your answers with those on page 23 of this lesson.
Correct any errors in your work.

EXERCISE 5 Factor each of the following trinomials. Check by multiplying.

1. $x^2 + 12x + 35$

Check:

2. $x^2 - 15x + 56$

Check:

3. $m^2 + 14m + 49$

Check:

4. $y^2 + 16y + 63$

Check:

$x + bx + c$ when $c < 0$

If $p \times q$ is negative the question is a little more complex.

To get a negative product, p and q must have different signs.

Example 1 Factor $y^2 - y - 12$.

$p \times q = -12$ and $p + q = -1$ ← *Remember: $-y = -1y$*

Possible Factors	$p \times q$	$p + q$	Do they fit?
-12, +1	$-12 \times 1 = -12$	$-12 + 1 = -11$	No
12, -1	$12 \times -1 = -12$	$12 - 1 = 11$	No
-6, 2	$-6 \times 2 = -12$	$-6 + 2 = -4$	No
6, -2	$6 \times -2 = -12$	$6 - 2 = 4$	No
4, -3	$4 \times -3 = -12$	$4 - 3 = 1$	No
-4, 3	$-4 \times 3 = -12$	$-4 + 3 = -1$	Yes

So, $p = -4$ and $q = 3$. You could also set $q = -4$ and $p = 3$.

The factors for $y^2 - y - 12$ are: $(y - 4)(y + 3)$.

Check: $(y - 4)(y + 3) = y^2 + 3y - 4y - 12$

$= y^2 - y - 12$

Most of the time you will not have to go through all of the factors of $p \times q$ in order. Very often the correct numbers will come to you immediately. Then, all you have to do is check them.

SELF-CORRECTING EXERCISE C

Factor each of the following trinomials.

1. $y^2 - 11y - 26$

2. $a^2 - 5a - 36$

3. $x^2 + 5x - 36$

4. $y^2 + y - 30$

Check your answers with those on page 24 of this lesson.
Correct any errors in your work.

EXERCISE 6 Factor each of the following trinomials. Check by multiplying.

1. $n^2 + 6n - 16$

Check:

2. $w^2 - 8w - 20$

Check:

3. $c^2 - c - 2$

Check:

4. $m^2 + 5m - 50$

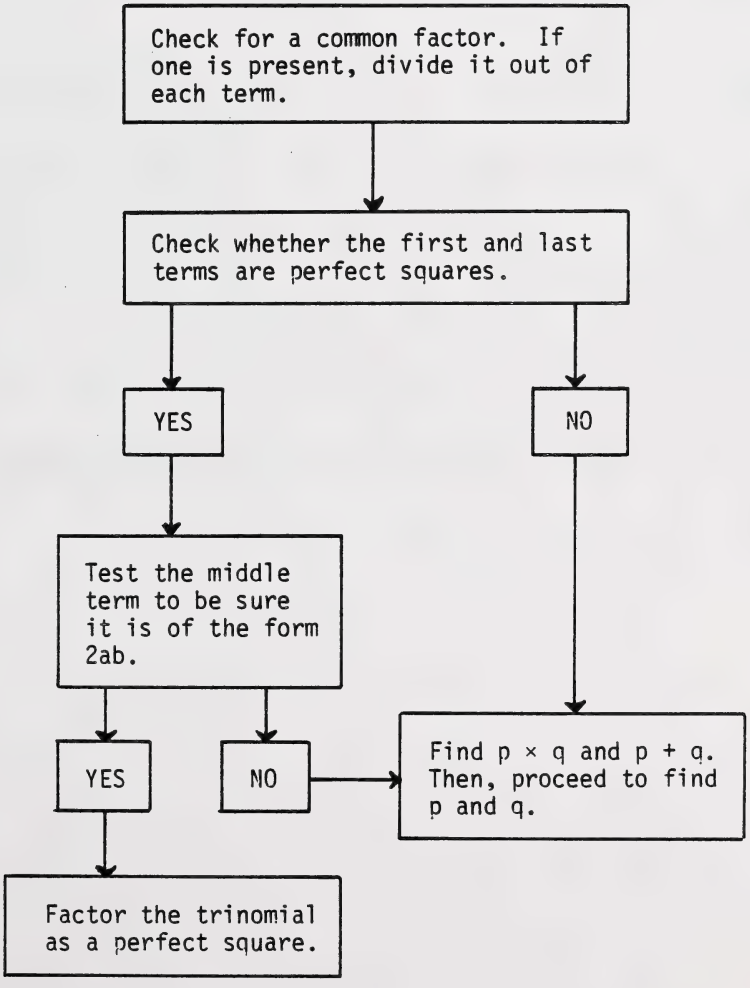
Check:

REVIEW OF TRINOMIAL FACTORIZATION

In Exercise 7 there is a mixture of all the types of trinomials you have factored in this lesson. Some are perfect squares and some are of the form $x^2 + bx + c$.

In some of the questions there is an added complication, a common factor which has to be taken out first.

Here, again, is the procedure:


Check for a common factor. If one is present, divide it out of each term.
Check whether the first and last terms are perfect squares.
YES
NO
Test the middle term to be sure it is of the form 2ab.
YES
NO
Find p × q and p + q. Then, proceed to find p and q.
Factor the trinomial as a perfect square.

Example 1 Factor $3x^2 + 9x + 6$.

$= 3(x^2 + 3x + 2)$ ← *Take out a common factor of 3.*

$p \times q = 2$

$p + q = 3$

$p = 2$, $q = 1$

$= 3(x + 2)(x + 1)$

Example 2 Factor $25a^3 - 40a^2 + 16a$.

$= a(25a^2 - 40a + 16)$ ← *Take out the common factor, a.*

Think: $25a^2 = 5a \times 5a$ $\quad 16 = 4 \times 4$

$2 \times 5a \times -4 = -40a$

This indicates a perfect square.

$= a(5a - 4)^2$

EXERCISE 7 Factor fully each of the following trinomials.

1. $x^2 - 5x - 6$

2. $x^2 - 5x + 6$

EXERCISE 7 (continued)

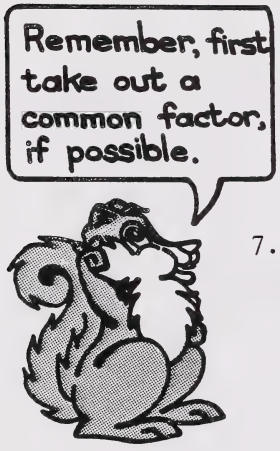
3. $9a^2 + 12a + 4$

4. $4a^2 + 20a + 25$

5. $25a^2 - 10a + 1$

EXERCISE 7 (continued)

6. $5x^2 - 5x - 100$



7. $4a^2 + 20a + 16$

8. $c^3 - 18c^2 + 81c$

KEY TO SELF-CORRECTING EXERCISES IN LESSON 27

SELF-CORRECTING EXERCISE A, page 8.

1. $a^2 - 10a + 25$

$$\sqrt{a^2} = a \text{ or } -a$$

$$\sqrt{25} = 5 \text{ or } -5$$

$$2 \times a \times -5 = -10a$$

$$a^2 - 10a + 25 = (a - 5)^2$$

2. $x^2 + 12x + 36$

$$\sqrt{x^2} = x \text{ or } -x$$

$$\sqrt{36} = 6 \text{ or } -6$$

$$2 \times x \times 6 = 12x$$

$$x^2 + 12x + 36 = (x + 6)^2$$

3. $4y^2 - 20y + 25$

$$\sqrt{4y^2} = 2y \text{ or } -2y$$

$$\sqrt{25} = 5 \text{ or } -5$$

$$2 \times 2y \times -5 = -20y$$

$$4y^2 - 20y + 25 = (2y - 5)^2$$

4. $144a^2 + 24a + 1$

$$\sqrt{144a^2} = 12a \text{ or } -12a$$

$$\sqrt{1} = 1 \text{ or } -1$$

$$2 \times 12a \times 1 = 24a$$

$$144a^2 + 24a + 1 = (12a + 1)^2$$

SELF-CORRECTING EXERCISE B, page 14.

1. $a^2 + 5a + 6$

$$\left.\begin{aligned} p + q &= 5 \\ p \times q &= 6 \end{aligned}\right\} \quad \begin{aligned} p &= 3 \\ q &= 2 \end{aligned}$$

$$a^2 + 5a + 6 = (a + 3)(a + 2)$$

2. $b^2 + 4b + 3$

$$\left.\begin{aligned} p + q &= 4 \\ p \times q &= 3 \end{aligned}\right\} \quad \begin{aligned} p &= 3 \\ q &= 1 \end{aligned}$$

$$b^2 + 4b + 3 = (b + 3)(b + 1)$$

3. $y^2 - 7y + 12$

$$\left.\begin{aligned} p + q &= -7 \\ p \times q &= 12 \end{aligned}\right\} \quad \begin{aligned} p &= -4 \\ q &= -3 \end{aligned}$$

$$y^2 - 7y + 12 = (y - 4)(y - 3)$$

4. $x^2 + 9x + 20$

$$\left.\begin{aligned} p + q &= 9 \\ p \times q &= 20 \end{aligned}\right\} \quad \begin{aligned} p &= 4 \\ q &= 5 \end{aligned}$$

$$x^2 + 9x + 20 = (x + 4)(x + 5)$$

SELF-CORRECTING EXERCISE C, page 17.

1. $y^2 - 11y - 26$

$$\left.\begin{array}{l} p \times q = -26 \\ p + q = -11 \end{array}\right\} \begin{array}{l} p = -13 \\ q = +2 \end{array}$$

$$y^2 - 11y - 26 = (y - 13)(y + 2)$$

2. $a^2 - 5a - 36$

$$\left.\begin{array}{l} p \times q = -36 \\ p + q = -5 \end{array}\right\} \begin{array}{l} p = -9 \\ q = +4 \end{array}$$

$$a^2 - 5a - 36 = (a - 9)(a + 4)$$

3. $x^2 + 5x - 36$

$$\left.\begin{array}{l} p \times q = -36 \\ p + q = +5 \end{array}\right\} \begin{array}{l} p = +9 \\ q = -4 \end{array}$$

$$x^2 + 5x - 36 = (x + 9)(x - 4)$$

4. $y^2 + y - 30$

$$\left.\begin{array}{l} p \times q = -30 \\ p + q = +1 \end{array}\right\} \begin{array}{l} p = 6 \\ q = -5 \end{array}$$

$$y^2 + y - 30 = (y + 6)(y - 5)$$

END OF LESSON 22

LESSON RECORD FORM

0902 Math 9
Revised 11/90

FOR STUDENT USE ONLY

Date Lesson Submitted

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Time Spent on Lesson

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(If label is missing or incorrect)

File Number

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Lesson Number _____

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(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

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Do **not** enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

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Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

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POLYNOMIAL OPERATIONS

In this lesson you will Review

- adding polynomials
- subtracting polynomials
- multiplying by monomials
- dividing by monomials
- multiplying binomials
- factoring polynomials

KEEPING SKILLFUL

EXERCISE 1 Perform each of the following operations.

1. $8(-3) =$

2. $16 \div (-4) =$

3. $-3(-2) =$

4. $-25 \div (-5) =$

5. $3(-2)(-1)(2)(-1) =$

6. $4(-3) \div (-6) =$

7. $(x^2)(x^3) =$

8. $(s^4)(s^3) =$

9. $m^5 \div m^4 =$

10. $y^3 \div y^3 =$

POLYNOMIALS

In Lesson 17 you learned of a mathematical object called a **polynomial**. The polynomial is a particular type of algebraic expression. It consists of a collection of terms, each with a numerical factor and whole number powers of one or more variables (letters).

You also learned that some polynomials have special names: monomial, binomial, trinomial.

In subsequent lessons you learned that polynomials can be added, subtracted, multiplied and divided. In fact, you can do almost anything to polynomials that can be done to numbers. The process is a little more complicated, but it can be done.

Reference

In this lesson you will again be performing the basic operations on polynomials. On page 51 of *Holt Mathematics 3* some of the properties of integers are listed. As you work on the polynomials, think about whether these properties apply to polynomials.

ADDITION AND SUBTRACTION

Addition

> To add polynomials, add like terms.

Subtraction

> To subtract a polynomial, change all of its signs and then add.

Example 1 $(2a + b) + (-3a + 2b)$

$= 2a + b - 3a + 2b$ *Remove brackets.*

$= 2a + (-3a) + b + 2b$ *Group like terms*

$= (2 + -3)\ a + (1 + 2)\ b$ *Combine like terms.*

$= -a + 3a$

Only like terms can be combined.

Example 2 $(2a + b) - (-3a + 2b)$

$= (2a + b) + (+3a - 2b)$ Change signs and add.

$= 2a + b + 3a - 2b$ Remove brackets.

$= 2a + 3a + b + -2b$ Group like terms.

$= (2 + 3)\ a + (1 + -2)\ b$ Combine like terms.

$= 5a - b$

Remember Like terms have the same letters (variables) and the letters have the same powers.

Example 3 From the sum of $5x^2 - 8x + 2$ and $3x^2 + 5x - 4$, subtract $8x^2 - 3x - 2$.

$(5x^2 - 8x + 2) + (3x^2 + 5x - 4) - (8x^2 - 3x - 2)$

$= (5x^2 - 8x + 2) + (3x^2 + 5x - 4) + (-8x^2 + 3x + 2)$

$= 5x^2 - 8x + 2 + 3x^2 + 5x - 4 - 8x^2 + 3x + 2$

$= 5x^2 + 3x^2 + (-8x^2) + (-8x) + 5x + 3x + 2 + (-4) + 2$

$= (5 + 3 + -8)\ x^2 + (-8 + 5 + 3)\ x + 2 + 2 + (-4)$

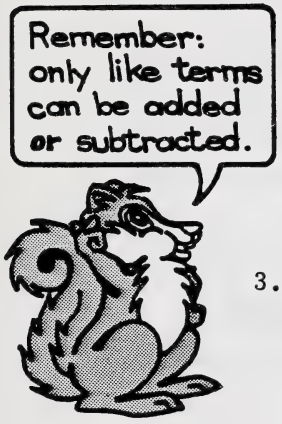
$= 0x + 0x + 0$

$= 0$

Exercise 2 1. Find the sum of $3n + 7$, $8n - 5$, $n + 6$ and $-4n + 9$.

EXERCISE 2 (continued)

2. Add $9x + 2y - z$, $-3x - 6y - 8z$ and $5x - 4y + 3z$.



3. Subtract $3n^2 + 8n - 21$ from $11n^2 - 14n - 7$.

4. From $d^2 - 12d + 36$, subtract $4d^2 - 9d$.

5. From the sum of $4c^2 - 10c + 8$ and $-5c^2 + 7c - 6$, subtract the polynomial $8c^2 - 6c + 2$.

MULTIPLICATION AND DIVISION BY MONOMIALS

The important thing to remember in multiplying or dividing a polynomial by a monomial, is that each term of the polynomial must be multiplied or divided separately.

Multiplication

> To multiply a polynomial by a monomial, multiply each term of the polynomial by the monomial.

Division

> To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

Examples

1. Multiply $6b$ by $d^2 - 5d - 3$.

$$6d(d^2 - 5d - 3)$$

$$= (6d)(d^2) + (6d)(-5d) + (6d)(-3)$$ ← *This line may be done in your head.*

$$= 6d^3 - 30d^2 - 18d$$

2. Divide $6n^3 + 12n^2 - 15n$ by $3n$.

$$(6n^3 + 12n^2 - 15n) \div (3n)$$

$$= \frac{6n^3}{3n} + \frac{12n^2}{3n} - \frac{15n}{3n}$$ ← *This line may be done in your head.*

$$= 2n^2 + 4n - 5$$

In arithmetic you checked your division answers by multiplying the quotient by the divisor. Do the same here.

$$3n(2n^2 + 4n - 5) = 6n^3 + 12n - 15n$$

Multiplication is the inverse operation of division. Thus, your result should be the original dividend.

3. Divide $14n^2 + 8n^3$ by $2n^2$ and check by multiplying.

$$\frac{14n^2}{2n^2} + \frac{8n^3}{2n^2} = 7 + 4n$$

Check by multiplying: $2n^2(7 + 4n) = 14n^2 + 8n^3$

EXERCISE 3

1. Multiply $6n^3 + 12n^2 - 15n$ by $3n$.

2. Divide $x^3y^3z^3 - 2x^2y^2z$ by xyz and check by multiplying.

3. $(20x^3y - 5x^2y^3) \div -5x^2y =$

MULTIPLYING BINOMIALS

In Lesson 21 you learned to multiply binomials using the FOIL rule. Remember that the FOIL rule is just a method used to make sure that you multiply each term in the first bonomial by each term in the second binomial.

Example 1 Multiply: $(3x + 2)(x - 1)$.

First	F	$(3x)(x) = 3x^2$
Outside	O	$(3x)(-1) = -3x$
Inside	I	$(+2)(x) = +2x$
Last	L	$(+2)(-1) = 2$

The sum is $-x$.

So, $(3x + 2)(x - 1) = 3x^2 - x - 2$.

Example 2 Multiply: $(6 - x)(2x - 1)$.

$= 12x - 6 - 2x^2 + x$

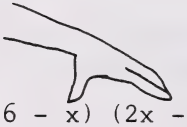
$= 12x + x - 6 - 2x^2$ Add like terms.

$= (12 + 1)x - 6 - 2x^2$

$= 13x - 6 - 2x^2$

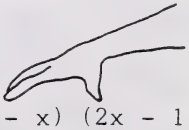
A good way to keep track of what you are multiplying is to put your thumb and index finger on the two terms being multiplied. With your other hand write down the product.

Then, move your thumb and finger to the next two terms to be multiplied. Continue until you have all the necessary products.



$(6 - x)(2x - 1)$

or, for left-handed people



$(6 - x)(2x - 1)$

Example 3 Multiply: $(a - 8)(2a + 5)$.

$$= 2a^2 + 5a - 16a - 40$$

$$= 2a^2 + (5 + -16)\,a - 40$$

$$= 2a^2 - 11a - 40$$

Exercise 4 Multiply. Use the finger method if you can.

1. $(x - y)(2x - y)$

2. $(2a + 3d)(a - 2d)$

3. $(2a + b)(4a - 5b)$

4. $(3m + 2n)(m + 3n)$

SPECIAL PRODUCTS

Now that you know how to multiply two binomials, you are ready to look at some special cases.

These are not really multiplied any differently, however, some of the answers are a bit different.

Example 1 Multiply $(2x - 3)^2$.

Remember that $x^2 = x \times x$ and $3^2 = 3 \times 3$.

$$(2x - 3)^2$$

$$= (2x - 3)(2x - 3)$$

$$= (2x)(2x) + (2x)(-3) + (-3)(2x) + (-3)(-3)$$

$$= 4x^2 - 6x - 6x + 9$$

$$= 4x^2 - 12x + 9$$

As long as you remember to write the square as a product you should have no problems.

Example 2 Multiply $(3a + 4b)(3a - 4b)$.

$$= (3a)(3a) + (3a)(-4b) + (+4b)(3a) + (+4b)(-4b)$$

$$= 9a^2 - 12ab + 12ab - 16b^2$$

Note: $-12ab + 12ab = 0$

$$= 9a^2 - 16b^2$$

In this case, the like terms added to 0.

Example 3 Multiply $(2a - 5)(3b + 2)$.

$$= (2a)(3b) + (2a)(+2) + (-5)(3b) + (-5)(+2)$$

$$= 6ab + 4a - 15b - 10$$

In this case, there were no like terms.

EXERCISE 5 Multiply.

1. $(a + 3b)(a - 3b)$

2. $(2x - y)(x + 2a)$

3. $(2c - 5)^2$

4. $(3x - 4y)(3x + 4y)$

5. $(a + b)^2$

FACTORING REVIEW

So far in this course you have learned four types of factoring along with their methods.

1. Common factoring
2. Difference of squares
3. Perfect square
4. Simple trinomial

Common Factoring

In Common Factoring, the Greatest Common Factor of the terms of the polynomial is found. Then, this GCF is divided out of each term.

Example 1 Factor $6xy - 3ab$.

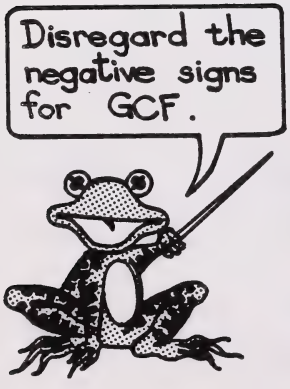
In this case, the GCF is obviously 3.

$$6xy - 3ab = 3\left(\frac{6xy}{3} - \frac{3ab}{3}\right)$$

$$= 3(2xy - ab)$$

Example 2 Factor $-6x^2y^5 + 8x^4y^3 - 2x^3y$.

Here the GCF is not so obvious. You might want to use a formal method to find it.



$$-6x^2y^5 = -2 \times 3 \times x \times x \times y \times y \times y \times y \times y$$

$$+8x^4y^3 = +2 \times 2 \times 2 \times x \times x \times x \times x \times y \times y \times y$$

$$-2x^3y = -2 \times x \times x \times x \times y$$

$$\text{GCF} = 2 \times x \times x \times y = 2x^2y$$

$$-6x^2y^5 + 8x^4y^3 - 2x^3y$$

$$= 2x^2y\left(\frac{-6x^2y^5}{2x^2y} + \frac{8x^4y^3}{2x^2y} - \frac{2x^3y}{2x^2y}\right)$$

$$= 2x^2y(-3y^4 + 4x^2y^2 - x)$$

EXERCISE 6 Factor.

1. $mx + my + 2mz$

2. $12b^2y - 3by^2$

3. $xy^2 - 2xy + 3x^2y$

4. $x^5 - x^2$

Difference of Squares

$$a^2 - b^2 = (a + b)(a - b)$$

To factor a difference of two squares:

1. Find the square root of the first term.
2. Find the square root of the last term.
3. Form the factors from the sum and difference of these square roots.

Example 1 Factor $16x^2 - 9y^2$.

Square root of $16x^2$ is $4x$.

Square root of $9y^2$ is $3y$.

This may be done in your head.

$16x^2 - 9y^2 = (4x + 3y)(4x - 3y)$

Example 2 Factor $1 - 25a^2$.

Square root of 1 is 1.

Square root of $25a^2$ is $5a$.

This may be done in your head.

$1 - 25a^2 = (1 + 5a)(1 - 5a)$

EXERCISE 7 Factor fully.

1. $64 - x^2$

2. $4x^2 - 1$

3. $9x^2 - 16$

4. $m^2k^2 - 4p^2$

Perfect Square $a^2 + 2ab + b^2 = (a + b)^2$

Factoring a perfect square consists of the following steps:

1. Find the square root of the first term.
2. Find the square root of the last term.
3. Check to see that the middle term is twice the product of the square roots from 1. and 2.
4. If the middle term checks out, form the factors from the square roots. Use the sign from the middle term.

Example 1 Factor $25w^2 - 40wy + 16y^2$.

Square root of $25w^2$ is $5w$.

Square root of $16y^2$ is $4y$.

$2 \times 5w \times 4y = 40wy$

$25w^2 - 40wy + 16y^2 = (5w - 4y)^2$

These signs are the same.

Example 2 Factor $16x^2 + 26x + 3$.

Square root of $16x^2$ is $4x$.

However, 3 has no simple square root. So, $16x^2 + 26x + 3$ is <u>not</u> a perfect square. Some other method of factoring must be used.

This trinomial can be factored, however, the method is beyond the scope of this course. It is only necessary for you to know that it is not a perfect square.

Example 3 Factor $4x^2 - 12x + 9$.

Square root of $4x^2$ is $2x$.

Square root of 9 is 3.

$2 \times 2x \times 3 = 12x$, which is the correct middle term.

So, $4x^2 - 12x + 9$ is a perfect square.

And, $4x^2 - 12x + 9 = (2x - 3)^2$

EXERCISE 8 Factor fully.

1. $x^2 + 4x + 4$

2. $m^2 - 24m + 144$

3. $x^2 + 8x + 16$

4. $16a^2 - 40a + 25$

Simple Trinomial

A simple trinomial is one in which the numerical coefficient of the first term is 1. The form looks like this:

$x^2 + bx + c$ *b and c are numbers.*

To factor $x^2 + bx + c$, it is necessary to find two numbers, p and q, such that:

$$p + q = b$$

$$p \times q = c$$

The following rules may help you:

1. If c is positive, then p and q will have the same signs. Both will be:
   - positive if b is positive, or,
   - negative if b is negative.

2. If c is negative, then p and q will have different signs. The larger number of p or q will have the same sign as b.

   Then $x^2 + bx + c = (x + p)(x + q)$.

Example 1 Factor $x^2 - 5x + 6$.

$p \times q = +6$ *p and q must have the same sign.*

$p + q = -5$ *p and q must be negative.*

Possible Factors	$p \times q$	$p + q$	Do they fit?
-6, -1	$-6 \times -1 = 6$	$-6 + -1 = -7$	No
-3, -2	$-3 \times -2 = 6$	$-3 + -2 = -5$	Yes

So, $p = -3$ and $q = -2$.

$x^2 - 5x + 6 = (x - 3)(x - 2)$

Example 2 Factor $x^2 - 5x - 6$.

$p \times q = -6$ *p and q have opposite signs.*

$p + q = -5$ *The larger number is negative.*

Possible factors. $-6 \times 1 = -6$ $-6 + 1 = -5$ *These two factors fit.*

Thus, $p = -6$, $q = 1$.

$x^2 - 5x - 6 = (x - 6)(x + 1)$

EXERCISE 9 Factor fully.

1. $y^2 - 3y - 70$

2. $x^2 + 14x + 48$

3. $x^2 - 12x + 35$

Mixed Methods

When factoring any algebraic expression always check first to see if there is a factor common to each term.

The second factor may be found by dividing the given expression by the common factor.

Then, check to see if the second factor is of one of the following types:

$a^2 - b^2$

$a^2 \pm 2ab + b^2$

$x^2 \pm bx + c$

± means plus or minus.

Use the appropriate method to factor.

Check again to be sure that none of the factors can be factored further.

Example 1 Factor $3x^2 - 3y^2$.

$3x^2 - 3y^2 = 3(x^2 - y^2)$ *Common factor is 3.*

$= 3(x + y)(x - y)$ *Difference of squares.*

Example 2 Factor $ax^2 + 6ax + 9a$.

$ax^2 + 6ax + 9a = a(x^2 + 6x + 9)$ *Now factor $x^2 + 6x + 9$*

$x^2 + 6x + 9 = (x + 3)^2$

So, $ax^2 + 6ax + 9a = a(x + 3)^2$

Example 3 Factor $2x^3 - 24x^2 - 26x$.

$2x^3 - 24x^2 - 26x = 2x(x^2 - 12x - 13)$ *Now factor $x^2 - 12x - 13$.*

$p \times q = -13$

$p + q = -12$

$-13 \times 1 = -13$, $-13 + 1 = -12$

So, $x^2 - 12x - 13 = (x - 13)(x + 1)$.

Thus, $2x^3 - 24x^2 - 26x = 2x(x - 13)(x + 1)$.

EXERCISE 10 **Factor fully.**

1. $3x^2 - 3x - 36$

2. $27x^2 - 48$

3. $p^2q^2 - 2pq^2 - 24q^2$

End of Lesson 23

LESSON RECORD FORM

0902 Math 9
Revised 11/90

FOR STUDENT USE ONLY

Date Lesson Submitted

_____

Time Spent on Lesson

_____

(If label is missing or incorrect)

File Number

_____

Lesson Number _____

FOR A.D.L.C. USE ONLY

Assigned Teacher: _____

Lesson Grading: _____

Additional Grading E/R/P Code: _____

Mark: _____

Graded by: _____

Assignment Code: _____

Date Lesson Received:

_____

Lesson Recorded _____

Student's Questions and Comments

Apply Lesson Label Here

Name

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Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

_____
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ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do **not** enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

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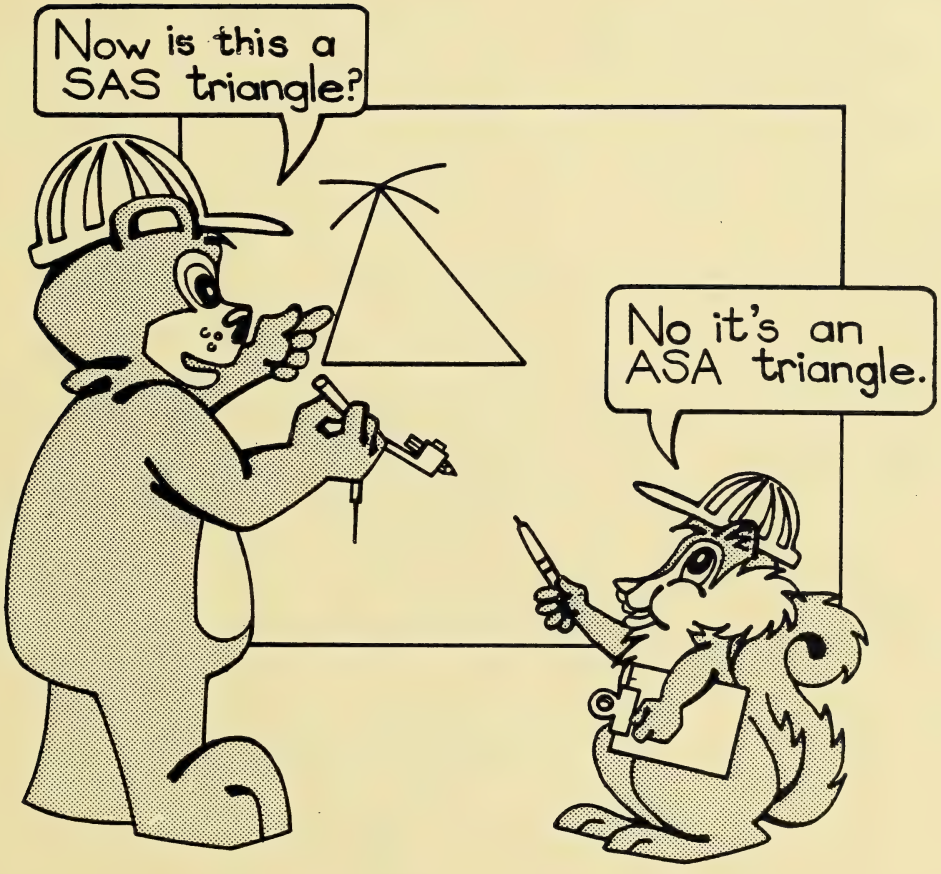
GEOMETRY



In this lesson you will learn about:

- types of angles and triangles
- constructing and bisecting segments and angles
- constructing three different kinds of triangles

KEEPING SKILLFUL

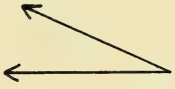
EXERCISE 1

1. Classify the following angles as being either acute, obtuse, straight or right. If you have forgotten the classifications, review pages 195 and 196 of *Holt Mathematics 3*.

(a)

(b)

(c)

(d) 

2. Measure the following angles with your protractor. Be as accurate as possible.

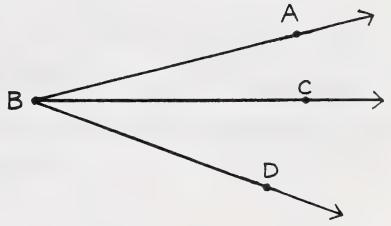
(a)

(b)

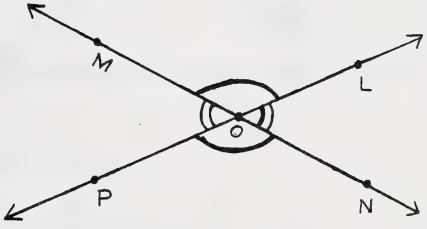
TYPES OF ANGLES AND TRIANGLES

Other types of angles not included on page 195 of *Holt Mathematics 3* are listed below.

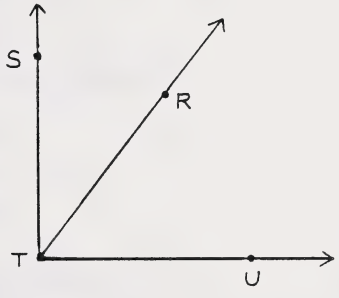
1. **Adjacent angles:** ∠ABC and ∠CBD are adjacent angles because they share a common ray, $\overrightarrow{BC}$, and a common vertex, B.



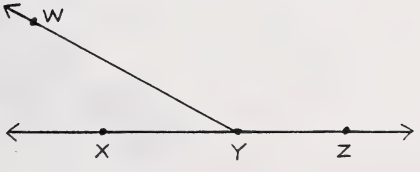
2. **Vertically opposite angles:** ∠MOL and ∠PON are vertically opposite angles. So are ∠MOP and ∠LON. Vertically opposite angles are congruent angles.



3. **Complementary angles:** ∠STR and ∠RTU are complementary angles because the sum of their measures is 90°.

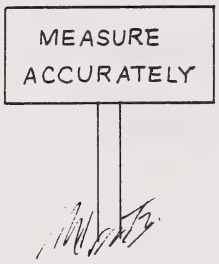


4. **Supplementary angles:** ∠WYX and ∠WYZ are supplementary angles because the sum of their measures is 180°.



EXERCISE 2 Do the following questions.

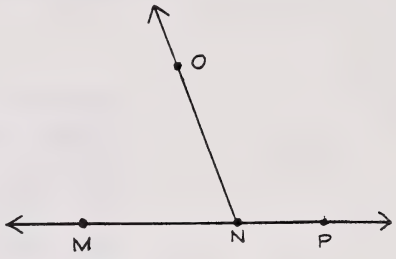
1. Use a protractor to measure the following angles.



(a) m ∠MNO = _____ °

m ∠ONP = _____ °

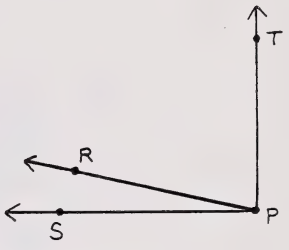
m ∠MNP = _____ °



(b) m ∠SPR = _____ °

m ∠RPT = _____ °

m ∠TPS = _____ °



2. State whether the following are true or false.

(a) When the measures of two supplementary angles are added, the result is equal to the measure of a straight angle. _____

(b) A scalene triangle has three congruent sides. _____

(c) An isosceles triangle does not have any congruent angles. _____

(d) A right angle must always measure 90°. _____

(e) An obtuse angle may measure 180°. _____

(f) Adjacent angles do not have to be congruent. _____

(g) There are three acute angles in an acute triangle. _____

(h) Vertically opposite angles must always be congruent. _____

(i) Two complementary angles must be acute. _____

CONSTRUCTING AND BISECTING SEGMENTS AND ANGLES

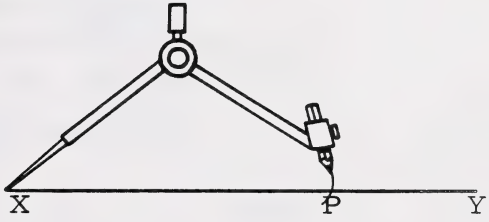
In constructing and bisecting segments you will use only a compass and a straight edge (or ruler).

Segments

To construct a segment equal to a given segment, first draw a segment longer than the given one. With your compass measure the given segment and, without changing the setting of your compass, cut your second segment.

Example

Given:



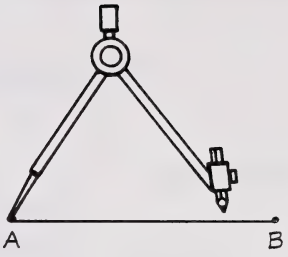
$\therefore \overline{AB} \equiv \overline{XP}$ $\equiv$ *means is congruent to.*

To check your construction, measure the length of $\overline{AB}$ and $\overline{XP}$. They should be equal.

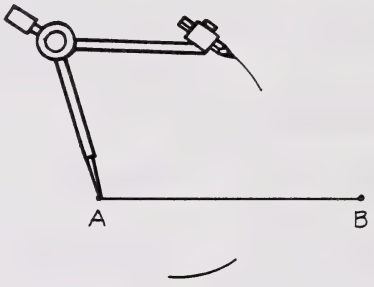
Bisecting Segments

To bisect $\overline{AB}$ follow these steps:

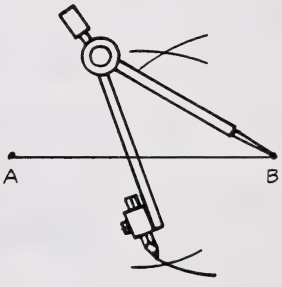
1. Open your compass so that the distance between the endpoint and the pencil point is over one-half the length of $\overline{AB}$.



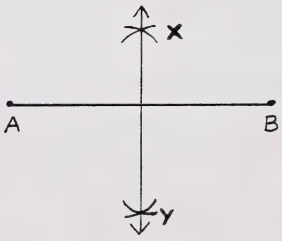
2. Place the compass point at A and draw two arcs (or one long arc) one above $\overline{AB}$ and one below $\overline{AB}$.



3. Without changing the setting of your compass, place the compass point at B and draw two arcs (or one long arc) to cut the former arcs.



4. Draw a line through the points where the arcs meet.

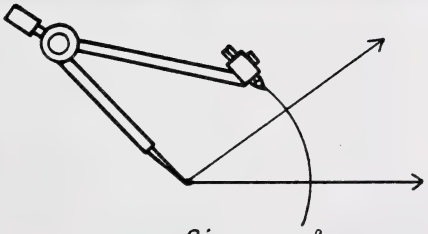


$\therefore$ $\overleftrightarrow{XY}$ bisects $\overline{AB}$.

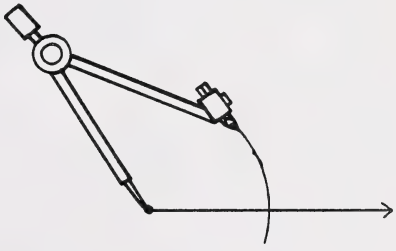
Angles

To construct an angle equal to a given angle follow these steps:

1. Draw a ray. Place your compass on the vertex of the given angle and make an arc cutting both of its rays. (Do not change the width of your compass.) Then place your compass at the endpoint of the drawn ray and draw an arc.

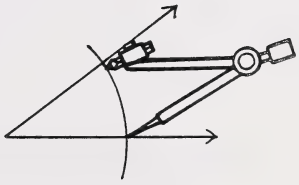


Given angle



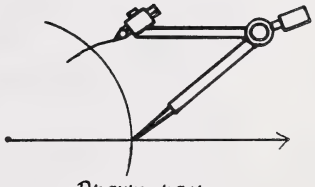
Drawn ray

2. Place your compass at the point, on the given angle, where the arc cuts one ray. Measure the distance to the point where this arc cuts the other ray.



Given angle

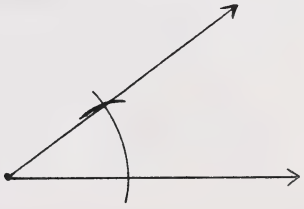
3. Keeping this distance on your compass, place the point of the compass on the drawn ray where the arc cuts it. Then, draw a second arc that cuts the first arc.



Drawn ray

4. Draw a ray from the endpoint through the point where the arcs intersect.

The constructed angle is now completed.

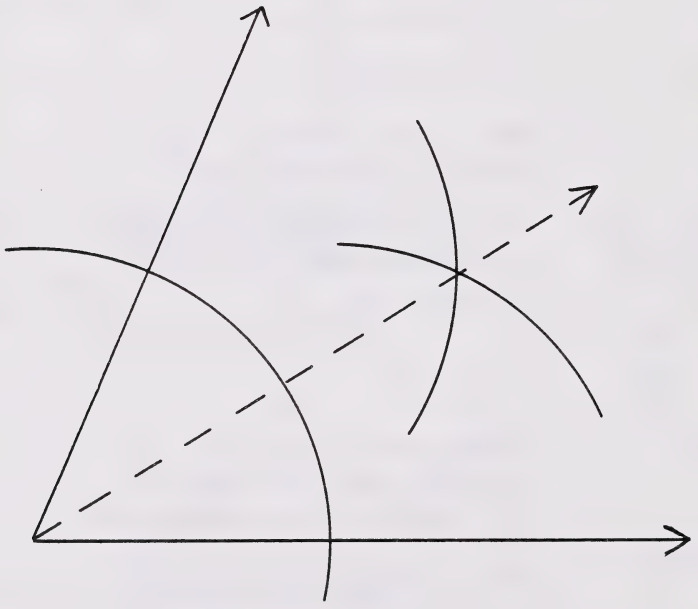


Reference

Study pages 168 and 170 of *Holt Mathematics 3*.

Bisecting Angles

1. To bisect a given angle first place the compass at the vertex and draw an arc to cut the two rays.
2. Place your compass point at each point where the arc cuts the rays. Draw two more arcs that intersect, keeping your compass at a constant radius.
3. Draw a ray from the vertex to the point where the two arcs intersect.



90° Angle

Constructing a 90° angle is the same as constructing a perpendicular to a line. There are two ways that 90° angles may be constructed:

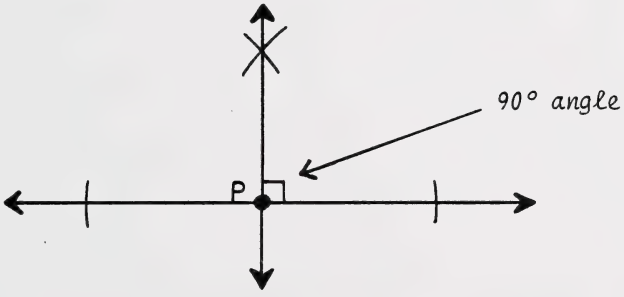
1. By constructing a perpendicular through a point on a line.
2. By constructing a perpendicular through a point outside the line.

Method 1:

(a) Chose a point P, on the line, and place your compass point on that point. Keeping your compass at a constant radius draw arcs cutting the line on both sides.

(b) Then place your compass at each point where the arc cuts the line. Keeping your compass at a constant radius, draw two other arcs, above or below the line, that intersect.

(c) Draw a line from the point P on the line to the point where the arcs intersect.

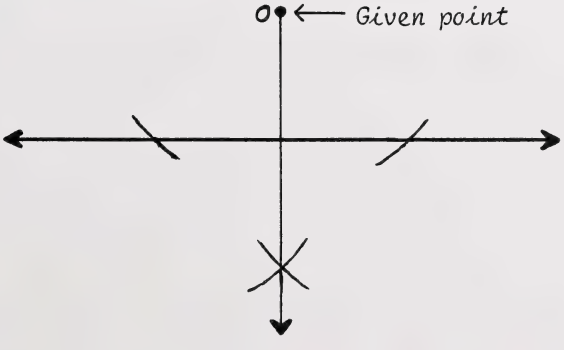


Method 2:

(a) Choose a point O not on the line. Place your compass point on that point and draw a arc to cut the line in two places.

(b) Place your compass point at each point where the arc cuts the line. Keeping your compass at a constant radius, draw two other arcs that intersect.

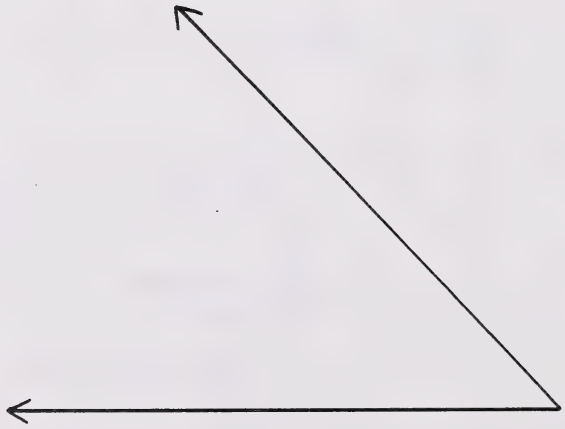
(c) Draw a line from the given point to the point where the two arcs intersect.



EXERCISE 3 Answer the following questions on segments and angles. Use only a compass and a straight edge.

1. Draw a segment 8 cm long and use a compass to bisect it.

2. Bisect the following angle. Show all construction arcs.



EXERCISE 3 (continued)

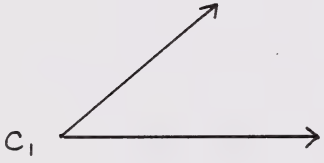
3. Construct a 90° angle and bisect it. Remember - you cannot use a protractor. Show all construction arcs.

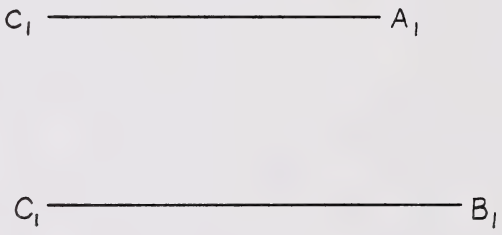
CONSTRUCTING TRIANGLES

Reference Study pages 200 to 202 of *Holt Mathematics 3.*

Part A Given two sides and the contained angle (SAS).

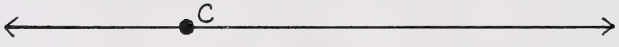
Example Construct $\triangle ABC$ so that $\angle C$, $\overline{CA}$ and $\overline{CB}$ are congruent with the following:



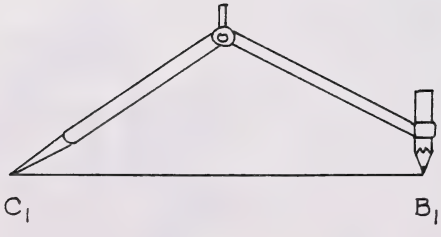


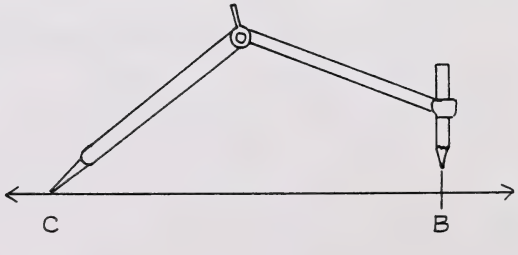
Method:

1. Draw a line on which to build the triangle and name the point C on it.

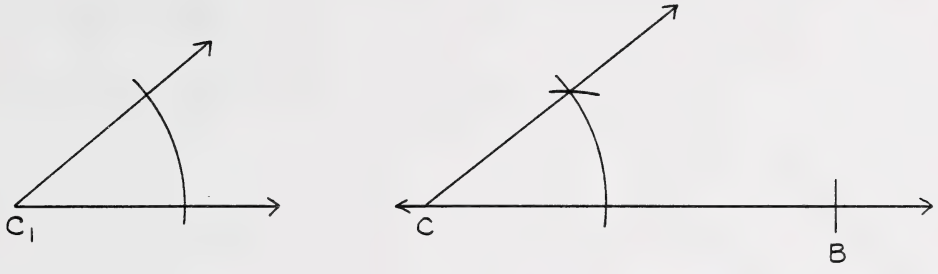


2. Use your compass to mark off $\overline{CB}$ equal to $\overline{C_1B_1}$.

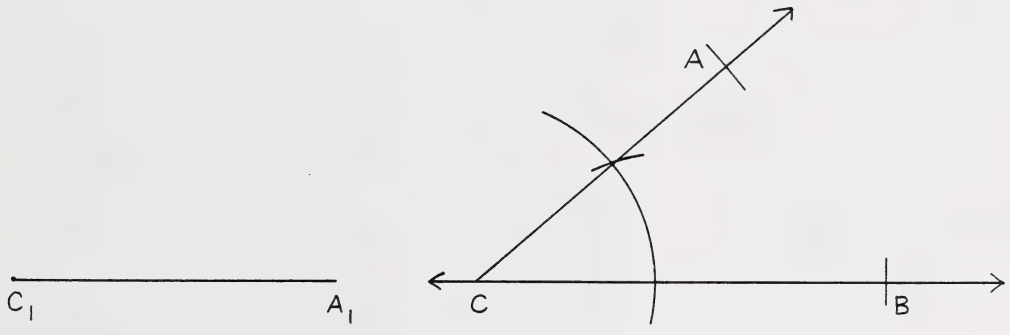




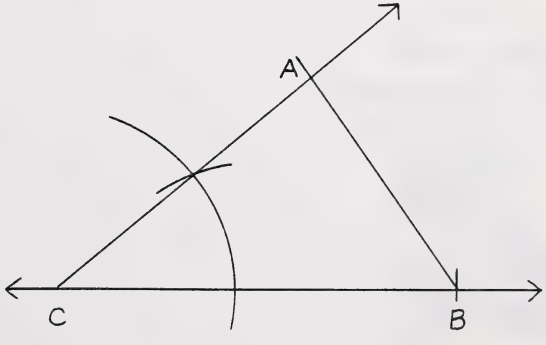
3. Construct an angle with its vertex at C that is congruent to $\angle C_1$.



4. Place your compass on point C and, on the ray you have drawn, mark off a line segment equal in length to $\overline{C_1A_1}$ to obtain point A.



5. Join A and B.



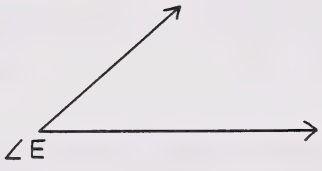
$\triangle ABC$ has been constructed.

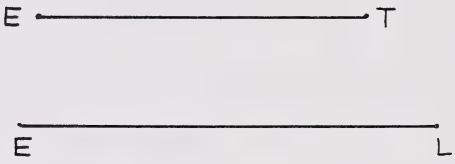
EXERCISE 4 Use only a compass and a straight edge for these constructions.

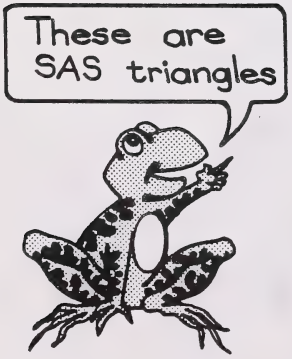
1. Construct $\triangle L_1 E_1 T_1$ so that: $\overline{ET} \cong \overline{E_1 T_1}$

$\overline{EL} \cong \overline{E_1 L_1}$

$\angle E \cong \angle E_1$

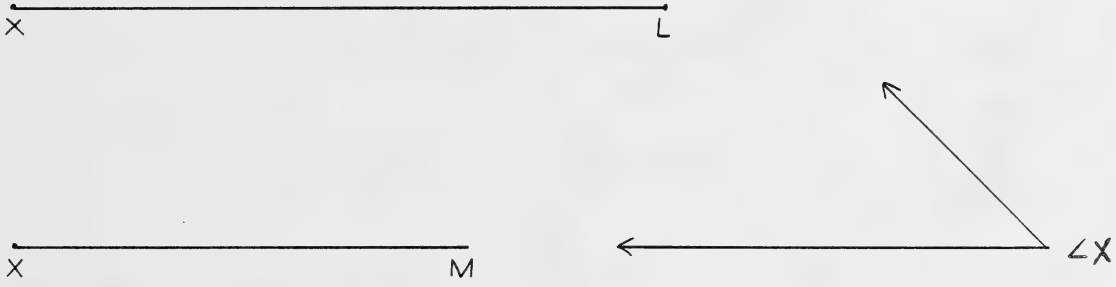






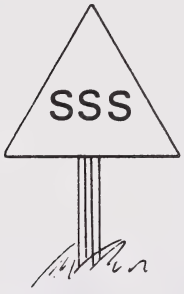
EXERCISE 4 (continued)

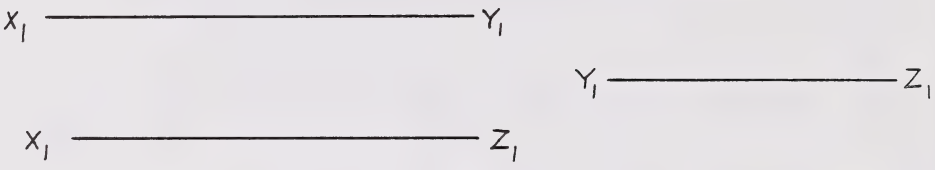
2. Given the measurements below, construct $\triangle XLM$. Show all construction arcs.



Part B Given three sides (SSS).

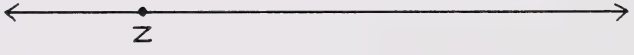
Example Construct $\triangle XYZ$ so that its sides are congruent to these segments:



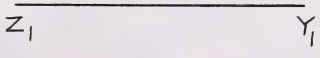


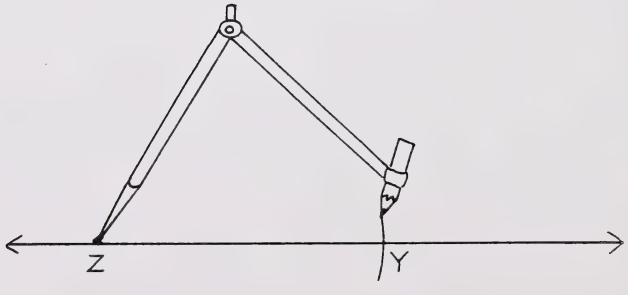
Method:

1. Draw a line on which to build the triangle and mark point Z on it.

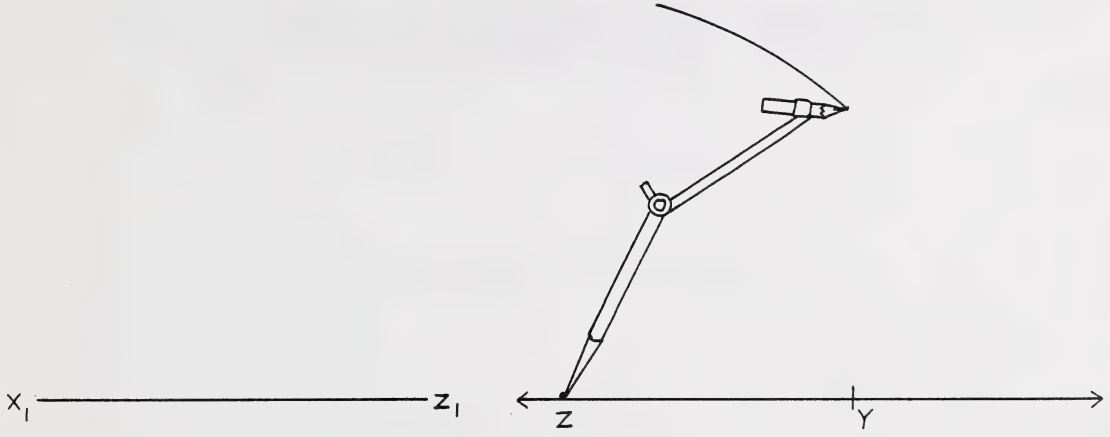


2. Mark off $\overline{ZY}$ so that $\overline{ZY} \equiv \overline{Z_1Y_1}$.

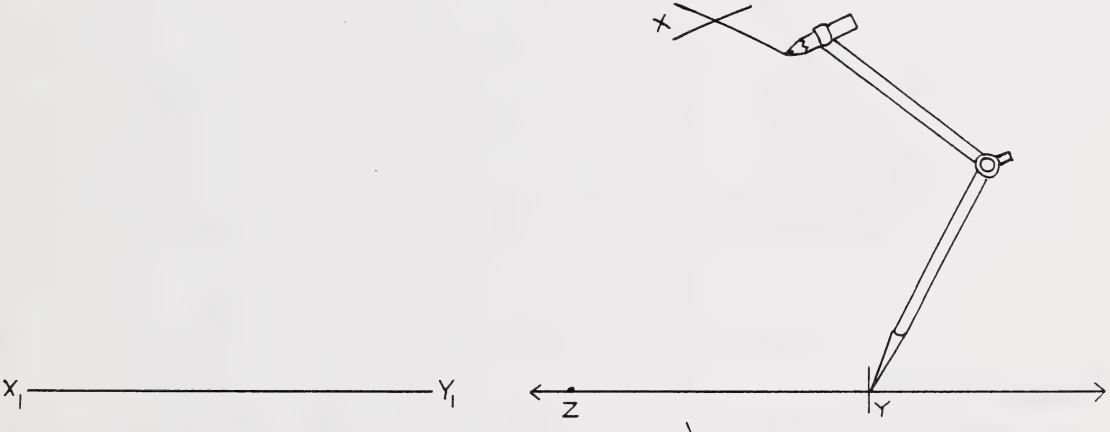




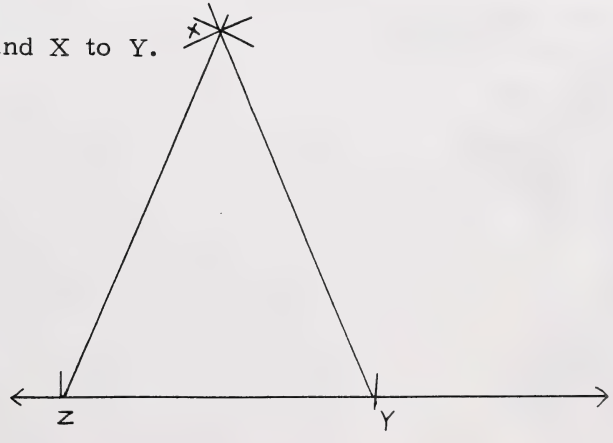
3. Open your compass to a distance equal to the length of $\overline{Z_1X_1}$. Put the metal tip on Z and draw an arc above the line.



4. Open your compass to a distance equal to the length of $\overline{X_1Y_1}$. With compass point on Y draw an arc to cut the first arc at X.



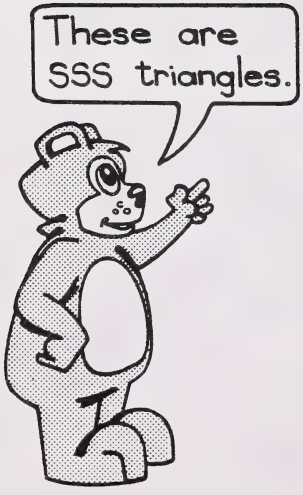
5. Join X to Z and X to Y.



$\triangle XYZ$ has been constructed.

EXERCISE 5 Use only compass and straight edge to do the following constructions.

1. Construct ΔTSU which has each side equal to the given length. Show all construction arcs.



EXERCISE 5 (continued)

2. Given the measurements below construct $\triangle ABC$. Show all construction arcs.

A ——————————— B

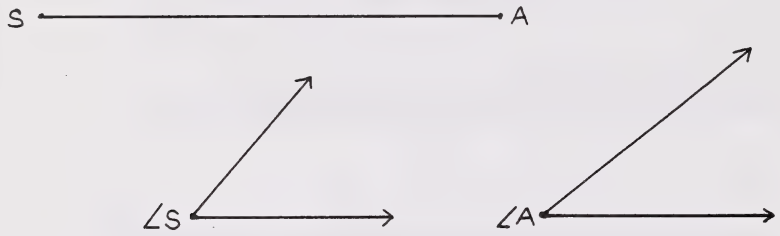
B ——————————————————— C

A ——————————————————————— C

Part C Given two angles and one side (ASA).

Reference See page 201 of *Holt Mathematics 3* for an example.

Example Construct △SAM given the following:

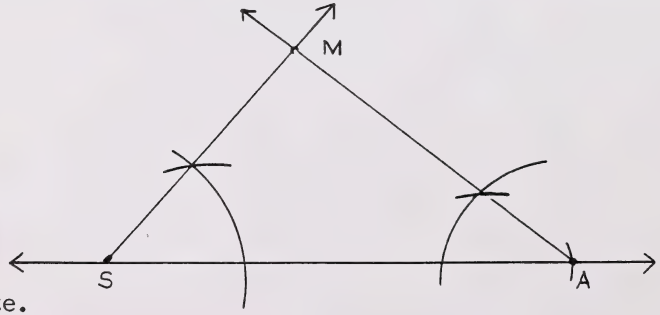


Method:

1. Draw a line on which to build the triangle and mark a point S on it.

2. With your compass measure $\overline{SA}$ above and mark off this segment on the line. Call the intersection point A.

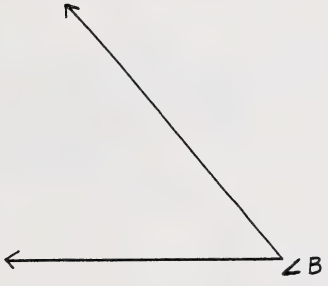
3. Now construct ∠S and ∠A on your line and label the point where the two rays meet M.

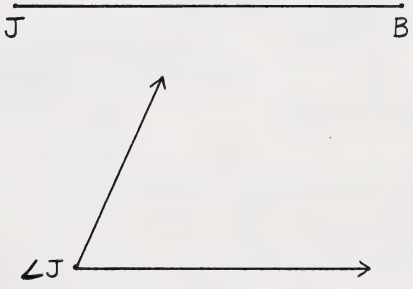


△SAM is now complete.

EXERCISE 6 Use compass and straight edge to do the following constructions. Show all construction arcs.

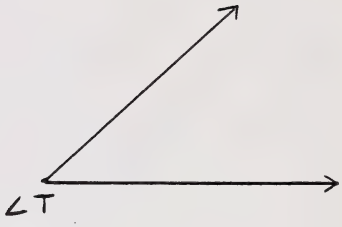
1. Construct $\triangle JOB$ given the following:





EXERCISE 6 (continued)

2. Construct $\triangle SPT$ given that angle $\angle P \equiv \angle T$ and given the following:



END OF LESSON 24

LESSON RECORD FORM

0902 Math 9
Revised 11/90

FOR STUDENT USE ONLY

Date Lesson Submitted

_____

Time Spent on Lesson

_____

(If label is missing or incorrect)

File Number

_____

Lesson Number _____

FOR A.D.L.C. USE ONLY

Assigned Teacher: _____

Lesson Grading: _____

Additional Grading E/R/P Code: _____

Mark: _____

Graded by: _____

Assignment Code: _____

Date Lesson Received:

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Lesson Recorded _____

Student's Questions and Comments

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Please verify that preprinted label is for correct course and lesson.

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1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do **not** enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

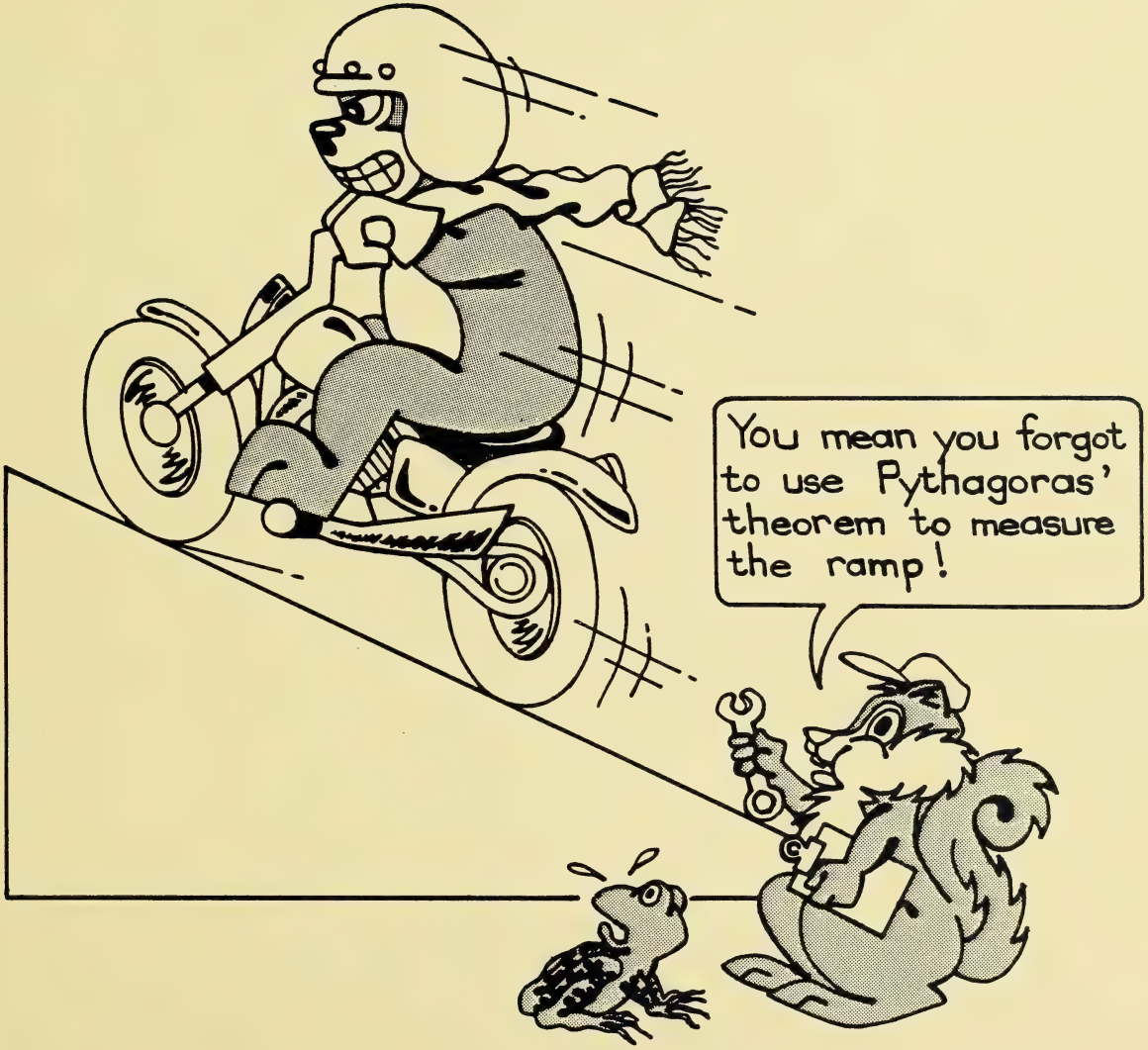
Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

St. Serv. 9-91

GEOMETRY



In this lesson you will learn about:

- constructing regular polygons
- using the Pythagorean Theorem
- using Heron's Formula

EXERCISE 1

1. Multiply $(2x - 3y)(3x + 4)$.

2. Divide $4x^2 - 10xy + 32xy^2$ by $-2x$.

3. Add $-3ax + 4x^2 + 3x$ and $-7x^2 + 4ax - 3x$.

CONSTRUCTING REGULAR POLYGONS

A regular polygon is a figure whose sides are the same length and whose angles are equal. A regular polygon with:

- 3 equal sides and angles is called an equilateral triangle.
- 4 equal sides and angles is called a square.
- 5 equal sides and angles is called a regular pentagon.
- 6 equal sides and angles is called a regular hexagon.

Reference

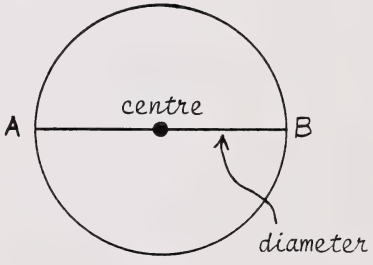
Study column one on page 197 of *Holt Mathematics 3* to find out how equilateral triangles are constructed.

Square

Follow these steps in constructing a square.

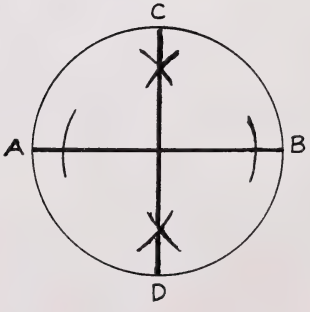
Step 1

Using a compass, construct a circle. Draw a diameter and label it $\overline{AB}$.



Step 2

Construct a perpendicular at the centre of the circle. Draw the perpendicular so that it touches both sides of the circle. Label these points C and D. (See Lesson 18 on constructing 90° angles or constructing bisectors.)



Step 3 Join points A, C, B, D.

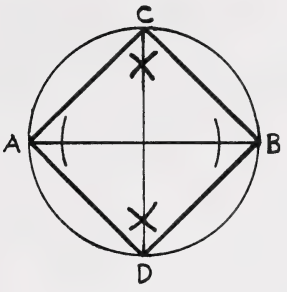
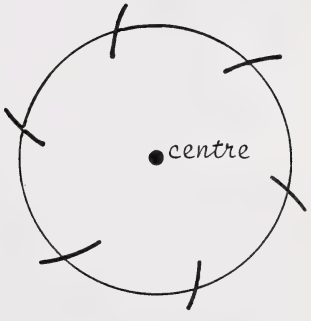


Figure ACBD is a square (regular polygon).

Hexagon Follow these steps in constructing a regular hexagon.

Step 1 Draw a circle with your compass.

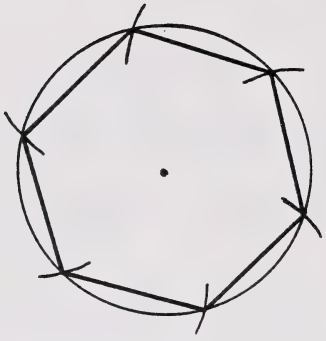
Step 2 Keeping the same radius, mark off six equal arcs on the circumference of the circle.



Reference Study Question 8 on page 197 of *Holt Mathematics 3.*

Step 3

Join the arcs on the circumference.



A regular hexagon has now been constructed.

EXERCISE 2

Do the following constructions using only a compass and a ruler. Be sure to show all the construction arcs.

1. Construct an equilateral triangle with each side 10 cm long.

EXERCISE 2 (continued)

2. Construct a regular hexagon. Use a radius of 3 cm for the circle.

3. Construct a square that has a diagonal 8 cm long.
Hint: The diagonal is the same as the diameter of a circle.

PERFECT SQUARES

> A perfect square is a number which can be written as a power with an exponent of 2.

4 is a perfect square, because it can be written as the power, 2^2. 16 is a perfect square because it can be written as the power, 4^2.

15 is not a perfect square, because it cannot be written as a power with an exponent of 2.

Study the Table of Squares on page 8. Notice the square of 10 and 20. For each zero in the base of the power there are two zeros in the square.

$10^2 = 100$

one zero *two zeros*

Example 1 Find the square of 170.

Note that 170 is made up of 17 times 10. The square of 17 (from the table) is 289.

When there is one zero in the number, the square will have two zeros.

$$17^2 = 289$$
$$170^2 = 28\ 900$$

The square of 170 is 28 900.

Example 2 Find the square of 900.

$$900^2 = 9^2 \times 100^2$$
$$= 81 \times 10\ 000$$
$$= 810\ 000$$

Two zeros in the base of the power result in four zeros in the square.

The square of 900 is 810 000.

Table of Squares of the Numbers 0 to 25

NUMBER	SQUARE OF NUMBER	FACTORS	PRODUCT (PERFECT SQUARE)
0	0^2	0×0	0
1	1^2	1×1	1
2	2^2	2×2	4
3	3^2	3×3	9
4	4^2	4×4	16
5	5^2	5×5	25
6	6^2	6×6	36
7	7^2	7×7	49
8	8^2	8×8	64
9	9^2	9×9	81
10	10^2	10×10	100
11	11^2	11×11	121
12	12^2	12×12	144
13	13^2	13×13	169
14	14^2	14×14	196
15	15^2	15×15	225
16	16^2	16×16	256
17	17^2	17×17	289
18	18^2	18×18	324
19	19^2	19×19	361
20	20^2	20×20	400
21	21^2	21×21	441
22	22^2	22×22	484
23	23^2	23×23	529
24	24^2	24×24	576
25	25^2	25×25	625

EXERCISE 3 Fill in the blanks.

1. (a) 16^2 = _____ (b) 40^2 = _____

   (c) 150^2 = _____ (d) 2400^2 = _____

2. For each of the following fill in the blank to indicate what number is being squared.

   (a) 81 = 9^2 (b) 8100 = 90^2

   (c) 400 = _____ (d) 441 = _____

   (e) 484 = _____ (f) 4 840 000 = _____

   (g) 62 500 = _____ (h) 1 960 000 = _____

SQUARE ROOT OF A PERFECT SQUARE

The square root of a number is indicated by the sign "$\sqrt{\ }$". Lesson 11 of this course has already dealt with finding square roots.

You should recall that every perfect square can be written as a power with an exponent of 2. Thus, you can easily find the square root of a perfect square.

Squaring a number and taking the square root are opposite operations. They cancel each other out.

$\sqrt{5^2}$ ← *opposite operations*

$= 5$

Turn over to page 10 and study the Table of Square Roots of Perfect Squares from 0 to 25.

Square Roots of Perfect Squares from 0 to 25

PERFECT SQUARE	PERFECT SQUARE AS A POWER	SQUARE ROOT
0	0^2	$\sqrt{0^2} = 0$
1	1^2	$\sqrt{1^2} = 1$
4	2^2	$\sqrt{2^2} = 2$
9	3^2	$\sqrt{3^2} = 3$
16	4^2	$\sqrt{4^2} = 4$
25	5^2	$\sqrt{5^2} = 5$
36	6^2	$\sqrt{6^2} = 6$
49	7^2	$\sqrt{7^2} = 7$
64	8^2	$\sqrt{8^2} = 8$
81	9^2	$\sqrt{9^2} = 9$
100	10^2	$\sqrt{10^2} = 10$
121	11^2	$\sqrt{11^2} = 11$
144	12^2	$\sqrt{12^2} = 12$
169	13^2	$\sqrt{13^2} = 13$
196	14^2	$\sqrt{14^2} = 14$
225	15^2	$\sqrt{15^2} = 15$
256	16^2	$\sqrt{16^2} = 16$
289	17^2	$\sqrt{17^2} = 17$
324	18^2	$\sqrt{18^2} = 18$
361	19^2	$\sqrt{19^2} = 19$
400	20^2	$\sqrt{20^2} = 20$
441	21^2	$\sqrt{21^2} = 21$
484	22^2	$\sqrt{22^2} = 22$
529	23^2	$\sqrt{23^2} = 23$
576	24^2	$\sqrt{24^2} = 24$
625	25^2	$\sqrt{25^2} = 25$

Example 1 Find the square root of 144.

$$\sqrt{144} = \sqrt{12^2}$$

$$= 12$$

The square root is the base of the power with the exponent of 2.

The square root of 144 ($\sqrt{144}$) is 12.

If an even number of zeros are attached to a perfect square, the resulting number will also be a perfect square. Thus:

- if the perfect square has two zeros, then the square root will have one zero.
- if the perfect square has four zeros, then the square root will have two zeros.

Rule

> The number of zeros in the square root of a number is one-half the number of zeros in the perfect square (using an even number of zeros).

Example 2 Find the square root of 1 960 000.

By referring to the table of square roots, you can find that the square root of 196 is 14.

Since there are four zeros in the perfect square, the square root will have two zeros.

$$1\,960\,000 = 196 \times 10\,000$$

$$\sqrt{1\,960\,000} = \sqrt{196} \times \sqrt{10\,000}$$

$$= 14 \times 100$$

$$= 1400$$

The square root of 1 960 000 is 1400.

EXERCISE 4 Find the following square roots.

1. (a) $\sqrt{4}$ = _____
   (b) $\sqrt{400}$ = _____
   (c) $\sqrt{40\ 000}$ = _____

2. (a) $\sqrt{25}$ = _____
   (b) $\sqrt{2500}$ = _____
   (c) $\sqrt{250\ 000}$ = _____

3. (a) $\sqrt{100}$ = _____
   (b) $\sqrt{10\ 000}$ = _____
   (c) $\sqrt{1\ 000\ 000}$ = _____

4. (a) $\sqrt{169}$ = _____
   (b) $\sqrt{16\ 900}$ = _____
   (c) $\sqrt{1\ 690\ 000}$ = _____

5. (a) $\sqrt{529}$ = _____
   (b) $\sqrt{44\ 100}$ = _____

6. (a) $\sqrt{625}$ = _____
   (b) $\sqrt{5\ 760\ 000}$ = _____

SQUARE ROOTS OF WHOLE NUMBERS THAT ARE NOT PERFECT SQUARES

Most whole numbers are not perfect squares. You may need to find the square root of such a number. A whole number, which is not a perfect square, cannot be written as a power with an exponent of two and a whole number as a base. Thus, it does not have an exact square root; its square root is only approximate.

To find the square root of some numbers that are not perfect squares refer to page 350 of *Holt Mathematics 3*.

EXERCISE 5 Use the table on page 350 of your textbook to find the approximate square roots of the following numbers.

1. $\sqrt{6}$ = _____
2. $\sqrt{10}$ = _____
3. $\sqrt{13}$ = _____
4. $\sqrt{26}$ = _____
5. $\sqrt{35}$ = _____
6. $\sqrt{47}$ = _____
7. $\sqrt{88}$ = _____
8. $\sqrt{72}$ = _____

THE PYTHAGOREAN THEOREM

In order to understand this theorem, it is necessary to review a right triangle.

Right Triangle

> A triangle with one right angle is called a right triangle.

If one angle of a triangle is 90°, then each of the other angles must be less than 90°. (If the other two angles were 90° or more, the sum of the three angles would be more than 180°.)

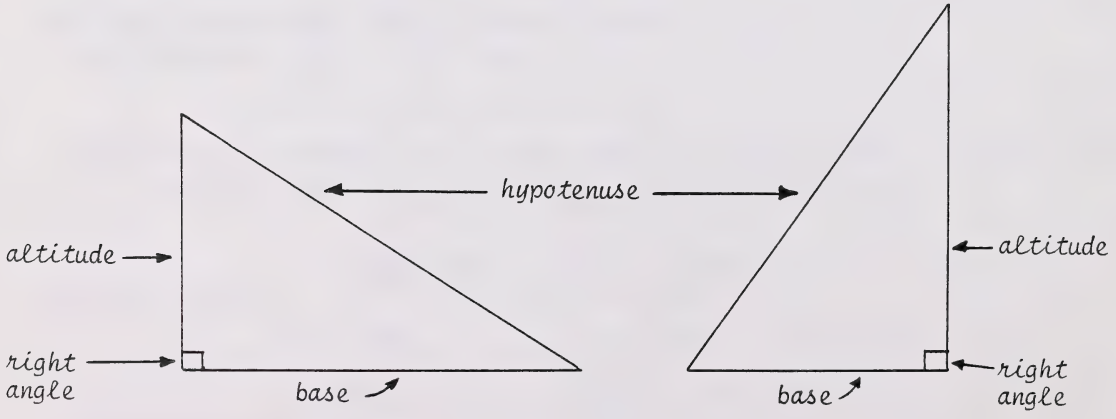
Every triangle has three sides. In a right triangle, these sides have been given special names.

Reference

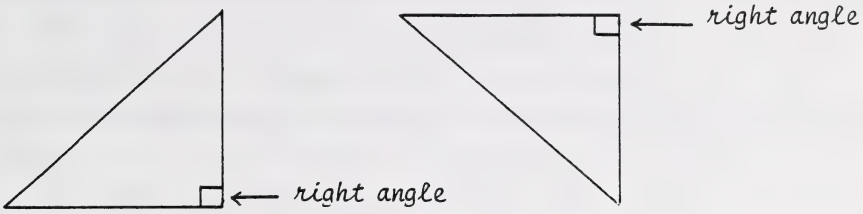
Study pages 94 to 100 of *Holt Mathematics 3*.

In a right triangle, the side opposite the right angle is the longest side and it is called the **hypotenuse**.

The other two sides that come together to form the right angle are called the **base** and the **altitude**. As the words indicate, the base is along the bottom of the triangle, and the altitude extends upwards from the base. The base can become the altitude, depending on which way you view the triangle.

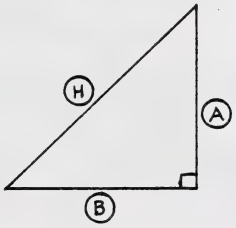


Note that you can indicate a right angle on a diagram by placing a little square in that angle.



EXERCISE 6 In each of the following triangles, mark the right angle with a little square and identify the hypotenuse with an "H", the altitude with an "A" and the base with a "B". Question one is done for you.

1.



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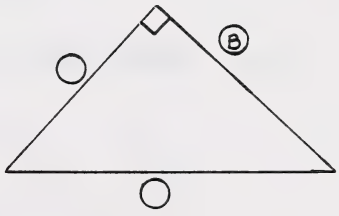

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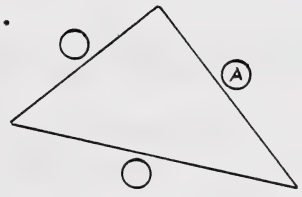

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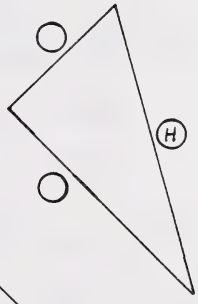

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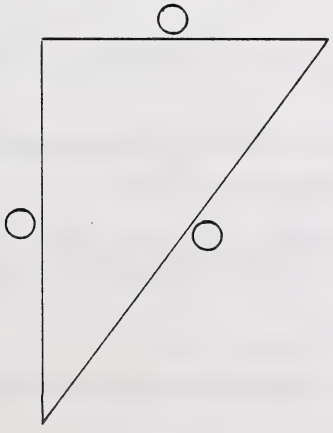

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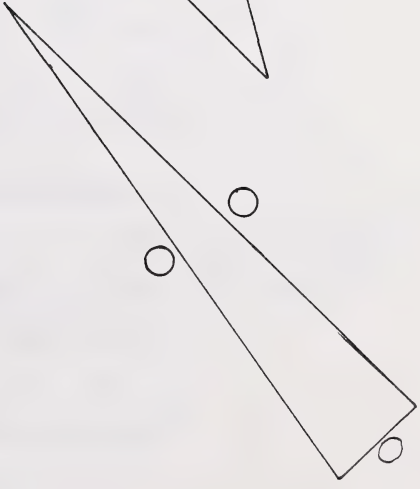
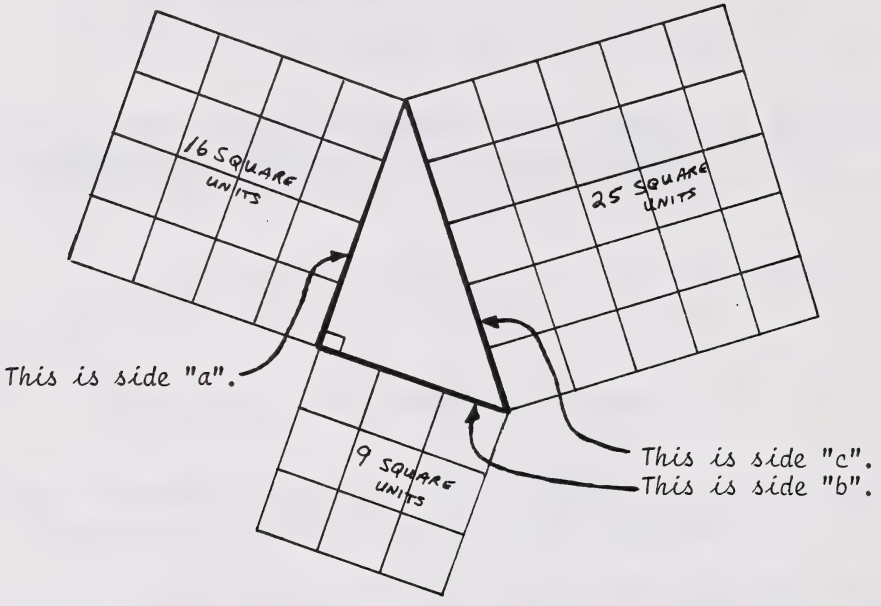


An ancient Greek mathematician, whose name was Pythagoras, discovered that there is a relationship between the hypotenuse of a right triangle and the other two sides of that same triangle. He found that if a square were drawn on each of the three sides of a right triangle, then the area of the square on the hypotenuse is equal to the sum of the area of the squares on the other two sides.



In this triangle the hypotenuse is labelled as side "c" and the other two sides are labelled "a" and "b".

The square of side "c" is 5^2 = 25 square units. The square on side "a" is 4^2 = 16 square units and on side "b" it is 3^2 = 9 square units.

Now, 25 = 16 + 9. That is $c^2 = a^2 + b^2$.

After drawing many such triangles, Pythagoras developed his famous theorem.

PYTHAGOREAN THEOREM

In any right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

$$c^2 = a^2 + b^2$$

It is customary to name the sides of a right triangle as in the diagram on page 15 and to express the Pythagorean Theorem as:

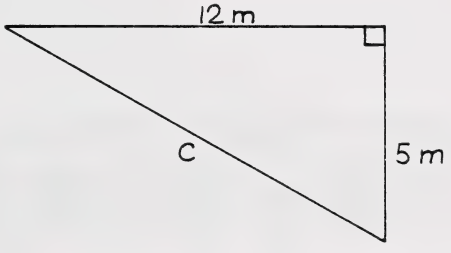
$$c^2 = a^2 + b^2$$

Now, how can you use this theorem? If you take the square root of each side of the equation you get:

$$c = \sqrt{a^2 + b^2}$$

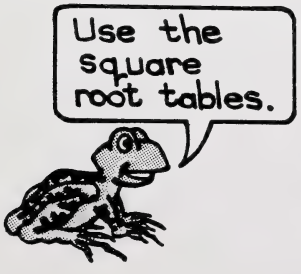
So, if you are given the length of the altitude and of the base of a right triangle, you can find the length of the hypotenuse. First, square the length of the altitude and of the base. Then, add those values together and take the square root of that sum.

Example 1 Find the length of the hypotenuse of the triangle below.



Use the Pythagorean Theorem; $c^2 = a^2 + b^2$.

The base (b) is 5 m and the altitude (a) is 12 m.



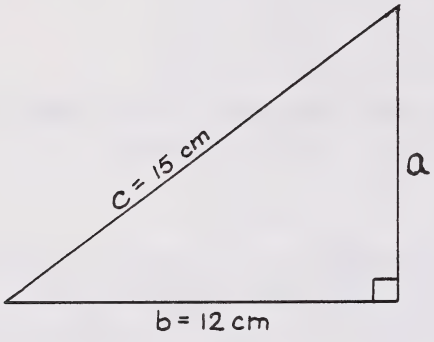
$$c^2 = 12^2 + 5^2$$
$$= 144 + 25$$
$$= 169$$

Since $c^2 = 169$, find the square root of 169 to obtain the value of c.

$$c = \sqrt{169}$$
$$c = 13$$

The hypotenuse is 13 m.

Example 2 Find the altitude of the triangle below.



$c = 15$ cm $\quad a = ?$ $\quad b = 12$ cm

$$c^2 = a^2 + b^2$$

$$15^2 = a^2 + 12^2$$

$$225 = a^2 + 144$$

$$225 - 144 = a^2$$

$$81 = a^2$$

$$\sqrt{81} = a$$

$$9 = a$$

The altitude of this triangle is 9 cm.

EXERCISE 7 Do the following questions using the Pythagorean Theorem.

1. Find the length of the hypotenuse of a right triangle if the base is 7 metres and the altitude is 24 metres.

   $c = ?$ $\quad a = 24$ m $\quad b = 7$ m

   The formula is $c^2 = a^2 + b^2$.

   $a^2 =$ _____

   $b^2 =$ _____

   $a^2 + b^2 =$ _____

   $c^2 =$ _____

   $c =$ _____

   The length of the hypotenuse is _____.

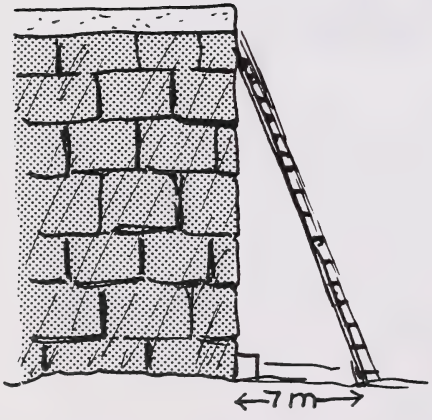
EXERCISE 7 (continued)

2. Find the length of the hypotenuse of a right triangle if the base is 8 dm and the altitude is 15 dm.

3. A room is 12 m long and 5 m wide. What is the length of the diagonal distance across the room?
Hint: draw a diagram to represent the problem.

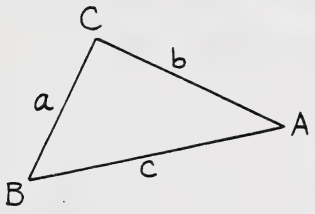
4. Find the length of the base of a right triangle if the altitude is 3 cm and the hypotenuse is 5 cm.

5. If a 25 m ladder is placed 7 m from a wall, how high up the wall can it reach?



FINDING AREA BY USING HERON'S FORMULA

Heron of Alexandria lived during the first century A.D. It has been mentioned that one day he was challenged to find the area of a triangle when only the measures of the three sides were known. Here is his solution:



If a, b, and c are the measures of the length of the sides of $\triangle ABC$, then the perimeter is a + b + c.

$$P = a + b + c$$

If s is one-half of the perimeter, then,

"s" is sometimes called the semi-perimeter.

$$s = \frac{a + b + c}{2}$$

Heron's formula is:

$$A = \sqrt{s(s - a)(s - b)(s - c)}$$

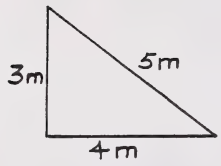
Reference

Study page 177, section C, of *Holt Mathematics 3.*

In finding the area of a triangle by Heron's method there are two steps to follow:

1. Find s or one-half of the perimeter.
2. Find A using Heron's formula.

Example 1 Find the area of the following triangle by using Heron's formula.



$$s = \frac{3 + 5 + 4}{2} = \frac{12}{2} = 6$$

$$A = \sqrt{6(6 - 3)(6 - 4)(6 - 5)}$$

$$= \sqrt{6 \times 3 \times 2 \times 1}$$

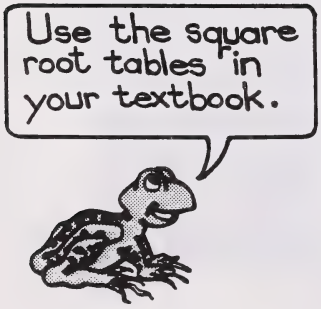
$$= \sqrt{36}$$

$$= 6$$

The area of the triangle is $6m^2$.

EXERCISE 8 Use Heron's formula in doing the following questions.

1. A triangular field has sides measuring 3 km, 5 km, and 6 km. Find its area.



EXERCISE 8 (continued)

2. The three sides of a triangular shaped resort lot are 5 km, 5 km, 6 km. Find the area of this lot using Heron's formula.

END OF LESSON 25

LESSON RECORD FORM

0902 Math 9
Revised 11/90

FOR STUDENT USE ONLY

Date Lesson Submitted

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Time Spent on Lesson

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(If label is missing or incorrect)

File Number

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1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do **not** enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

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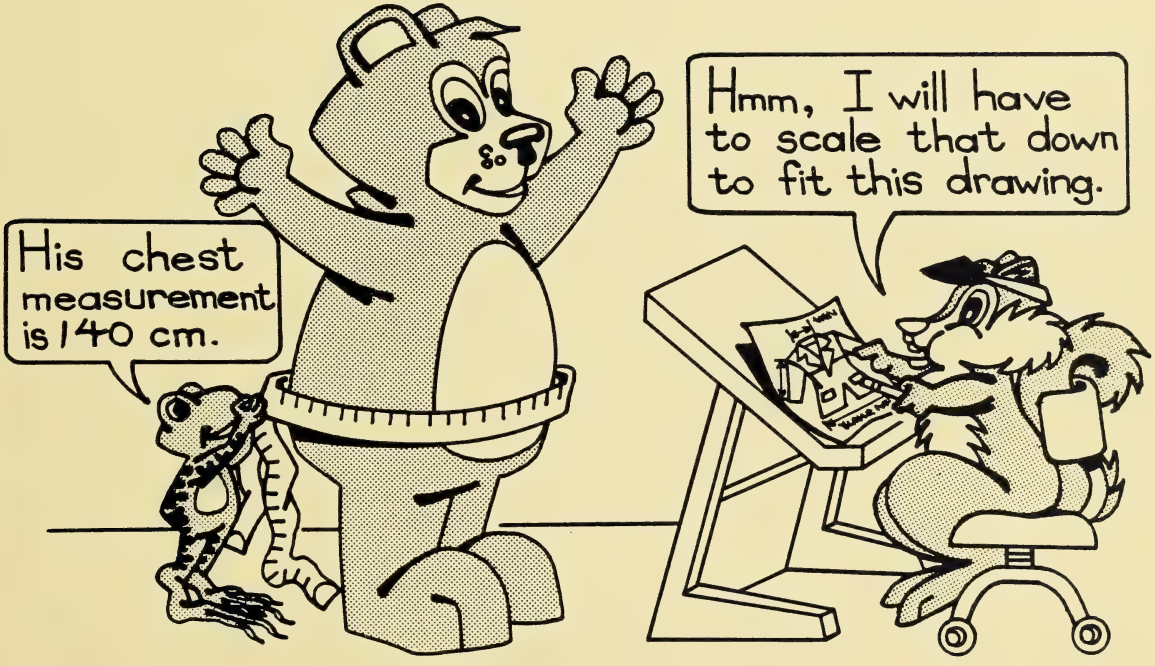
PROPORTIONS



In this lesson you will learn about using ratios in:

- distance, speed and time problems
- scale drawings

KEEPING SKILLFUL

EXERCISE 1 1. Find the unknown by using cross-multiplication.

(a) $\frac{19}{18} = \frac{w}{27}$

(b) $\frac{12.2}{N} = \frac{9}{4.5}$

(c) $\frac{3\frac{1}{2}}{5\frac{1}{4}} = \frac{3\frac{1}{3}}{y}$

(d) $\frac{5}{4.8} = \frac{2\frac{1}{2}}{w}$

DISTANCE, SPEED, TIME

The relationship between distance, speed and time is called motion. Motion is the result of an object travelling at a certain rate of speed for a certain amount of time. The object travels a certain distance. Some practical examples are:

- The fastest flight from Gander, Newfoundland to Belfast was made in 4 h 18 min 24.4 s.
- A train in Japan, between Tokyo and Osaka, travels at an average speed of 160 km/h.
- The fastest animal, the cheetah, has been clocked at 110 km/h.

In each of the above examples there is distance, speed and time. The relationship between these three items may be expressed as a formula.

Formula

Distance = speed(or velocity) × time

The abbreviation for distance is d, for speed is v and for time is t.

So then the formula would be:

$$d = vt$$

To find speed, the relationship would be:

$$v = \frac{d}{t}$$

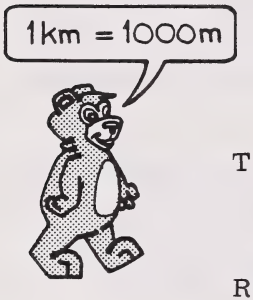
To find time, the relationship would be:

$$t = \frac{d}{v}$$

Changing Units

When dealing with distance, speed and time it is important to remember which units to use. Sometimes it may be necessary to change from metres to kilometres, from seconds to hours and so on.

Example 1 Roy drives a car at 10 m/s. How fast is this in km/h?



10 m/s means 10 metres per second.
So, in 1 minute, the speed would be $10 \times 60 = 600$ m/min.
In 1 hour it would be $600 \times 60 = 36\ 000$ m/h.

To find the speed in km/h, divide by 1000.

$36\ 000 \div 1000 = 36$ km/h

Roy is driving his car at 36 km/h.

EXERCISE 2 Do the following questions involving changing units. Show your work.

1. Find the speed in km/h.

   (a) 30 m/s (b) 500 m/min

2. Which is faster: a horse running at 7.5 m/s or a sparrow flying at 25 km/h?

Example 2 Pedro runs at the rate of 6 m/s. How far will he run in 5 minutes? (m/s means metres per second)

The correct formula to use would be $d = vt$ since the distance is what must be found.

Before calculations can be done, the rate of speed must be changed from seconds to minutes.

So, $6 \text{ m/s} = 6 \times 60 = 360 \text{ m/min}$

Now, $d = vt$

$$d = \frac{360 \text{ m}}{\cancel{\text{min}}} \times 5 \cancel{\text{min}}$$

$$= 1800 \text{ m}$$

Pedro will run 1800 m in 5 minutes.

Example 3 A car travels 900 km in 9.5 hours. What is its average speed?

$d = 900 \text{ km}$ $v = ?$ $t = 9.5 \text{ h}$

Then, $v = \frac{d}{t}$

$$v = \frac{900 \text{ km}}{9.5 \text{ h}}$$

$$v = 94.7 \text{ km/h}$$

$$9.5\overline{)900.0} \quad 94.7$$
$$855$$
$$450$$
$$380$$
$$700$$
$$665$$
$$35$$

The car travels at an average rate of 94.7 km/h.

EXERCISE 3 Use the formulas for distance, speed and time to answer the following questions.

1. An airplane travels at 450 km/h. How far will it travel in:

   (a) 6 hours? (b) 45 minutes?

2. A car averages 80 km/h. How long will it take this car to go:

   (a) 760 km? (b) 1900 m?

3. (a) Mark Spitz swam the 100 m freestyle in 51.2 seconds. What was his speed? (Round your answer to the nearer hundredth.)

3. (b) Renate Stecher of East Germany ran 100 m in 11.1 seconds. What was her speed? (Round your answer to the nearer tenth.)

Variables

So far you have used whole numbers and decimal numbers to represent the distance, speed or time. Now you will learn to use variables as replacements for distance or speed or time.

> A variable is a letter used to represent a number.

A common letter that is used to represent a number is x. However, any letter may be used.

At times you may wish to draw diagrams to help you understand a problem clearly. (Note Example 2 on page 8.)

Example 1

A bus travels at $(y + 1)$ km/h. How far will it travel in 14 hours?

$d = ?$ km $\qquad v = (y + 1)$ km/h $\qquad t = 14$ h

$$d = vt$$

$$d = \frac{(y + 1)\text{km}}{\text{h}} \times 14 \text{ h}$$

$$d = 14(y + 1)\text{km}$$

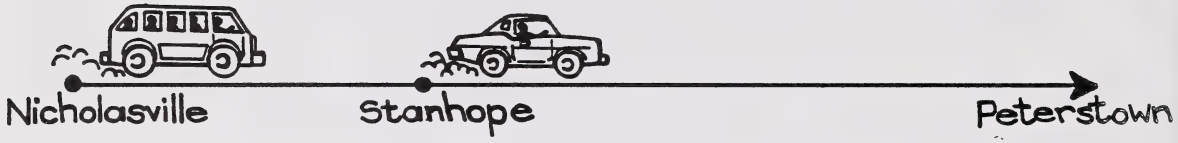
The distance the bus travels is $14(y + 1)$ km.

Example 2 Halvor takes a trip by bus from Nicholasville to Stanhope and then takes a car from Stanhope to Peterstown. How long is the total trip?

See the information and the diagram below.

By bus: speed = 90 km/h
time = (n - 3)h

By car: speed = 70 km/h
time = (n + 3)h



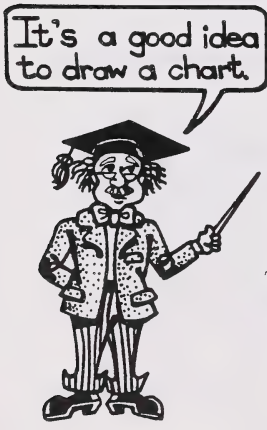
To find the total distance, you will first have to find the distance the bus travels and then the distance the car travels.

Distance by bus = 90 x (n-3)

d = vt

Distance by car = 70 x (n + 3)

It may help to set up a chart like this:



	d	v	t
Bus	?	90	n - 3
Car	?	70	n + 3

Total distance = distance by bus + distance by car

= 90 x (n - 3) + 70 x (n + 3)

= 90n - 270 + 70n + 210

= 160n - 270 + 210

= 160n - 60

The total distance Halvor travels is (160n - 60) km.

APPLICATIONS

EXERCISE 4 You may find that drawing diagrams for the following problems will be helpful.

1. A bus travelled through the mountains for 4 h and then travelled through the prairies for 5 h. The total distance travelled was 800 km. If the bus went 20 km/h faster through the prairies, what is the speed through the mountains?

Let x represent the speed in the mountains.

Mountains: *speed* = x
time = 4
distance = ?

Prairies: *speed* = x + 20
time = 5
distance = ?

Total Distance = *Distance through Mountains* + *Distance through Prairies*

	d	v	t
Mountains		x	4
Prairies		x + 20	5

Total Distance = *Distance through Mountains* + *Distance through Prairies*

$800 = 4x + 5(x + 20)$

=

=

=

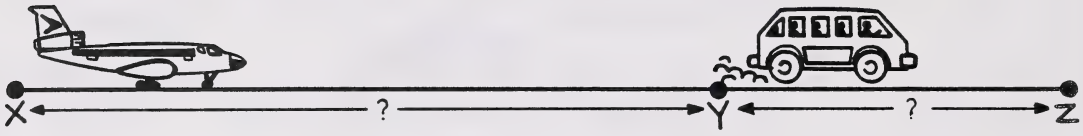
$= x$

EXERCISE 4 (continued)

2. The following trip was taken first by plane and then by bus. Find the total distance of the trip.

By plane: speed = 200 km/h
time = (9x + 2) h
distance = ?

By bus: speed = 80 km/h
time = (2x - 4)h
distance = ?



Chart

	d	v	t
plane		200	9x + 2
bus		80	2x - 4

Total distance = Distance by plane + Distance by bus

=

EXERCISE 5 (continued)

3. Brian and Jon live in two towns 600 km apart. On Saturday they left at the same time and drove toward each other. Brian drove at 100 km/h while Jon drove at 60 km/h. How much time will pass before they meet on the highway? (Make a diagram)

Brian:
Time
Speed
Distance

Jon:
Time
Speed
Distance

	d	v	t
Brian			
Jon			

Hint: The distance Brian has travelled plus the distance Jon has travelled is 600 km.

SCALE DRAWINGS

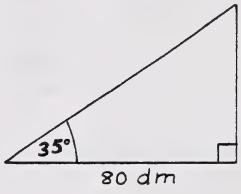
When an architect designs a house, he cannot make the plans "full size". Instead he draws a smaller picture of the house, much like a camera makes a photograph, which is smaller than the original object. In an atlas maps are drawn to scale. The scale for a map may be, 1 cm represents 100 km.

Sometimes a scale drawing may be larger than the actual object. In biology, drawings of an insect like an ant are made larger, so the small details can be shown. Here the scale may be, 1 cm represents 1 mm.

A scale drawing is in proportion to the real object. The diagram is exactly like the object except it is smaller or larger.

Example Alex wanted to make a drawing of the triangular sail of his boat.

The rough sketch at the right gives the length of one side and the size of two angles.



Make a scale drawing of the sail. Use the scale of 1 cm represents 5 dm.

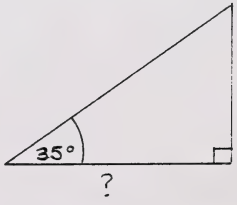
You know the ratio is 1 cm to 5 dm. With this ratio, you can set up a proportion to find how long 80 dm will be in your drawing.

Let n be the length in cm which represents 80 dm.

$$\begin{matrix} cm \longrightarrow \\ dm \longrightarrow \end{matrix} \frac{1}{5} = \frac{n}{80} \begin{matrix} \longleftarrow cm \\ \longleftarrow dm \end{matrix}$$

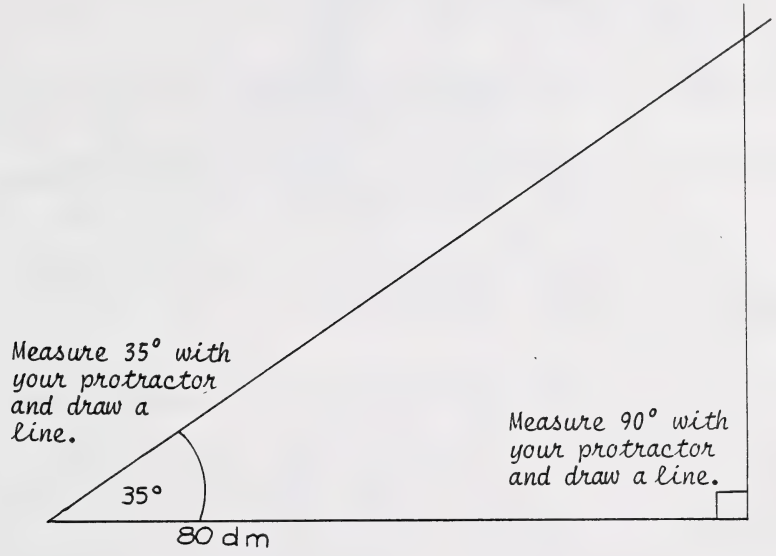
$$5n = 80$$

$$n = 16$$



The 80 dm would be represented by 16 cm in the drawing.

Now you have the size of two angles and the length of the included side (in cm). You can make a scale drawing representing the sail. The drawing has a scale of 1 cm to 5 dm.



EXERCISE 6 1. Make a scale drawing of the triangle using the given scale.

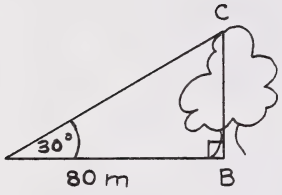
Linda is designing a large triangular pennant for her classroom. The sides of the pennant measure 6 m, 8 m and 10 m. She made a drawing of the pennant using the scale of 1 cm to 1 m. Make the scale drawing.

SCALE DRAWINGS AID IN MEASURING

Scale drawings help you to measure distances that would be difficult or impossible to measure with a ruler or a tape measure.

Example

Find the height of a tall tree. Use the facts you know from the measurements given in the rough sketch.



You want to make a scale drawing. A convenient scale to use would be 1 cm to 10 m.

First, you have to find the length you would draw to represent 80 m.

Variable

Let t be the length in cm to represent 80 m.

Equation

$$\frac{1}{10} = \frac{t}{80}$$ ← *Set up a ratio.*

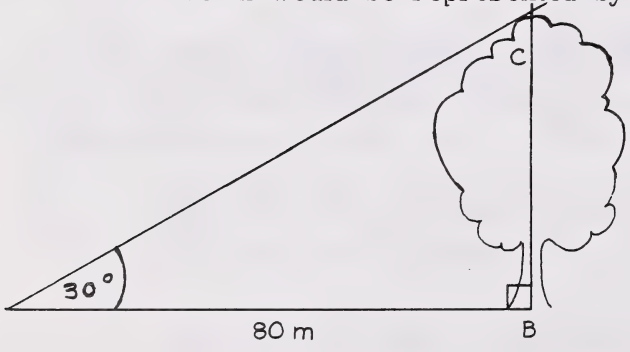
Solution

$$80 = 10t$$ ← *Cross multiply.*

$$8 = t$$

Statement

80 m would be represented by 8 cm.



You now have the length of one side and two adjacent angles. The drawing should look like the one at the left.

With a ruler, measure $\overline{CB}$.

$\overline{CB}$ is 4.5 cm long.

Variable

To find the height of the tree, let x represent the height.

Equation

$$\frac{1}{10} = \frac{4.5}{x}$$ ← *Set up a ratio.*

Solution

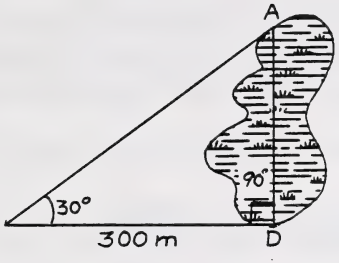
$$10 \times 4.5 = x$$ ← *Cross multiply.*

$$45 = x$$

Statement

The height of the tree is 45 m.

Exercise 7 1. Find the missing measurement by using a scale drawing.



The rough sketch to the left has the measurements the surveyors made near a swamp.

Find the distance, $\overline{AD}$, across the swamp.

Use the scale of 1 cm to 30 m.

Variable:

Equation

Solution

Statement

Scale Drawing

Variable

Equation

Solution

Statement

FINDING DISTANCES ON MAPS

The actual size of an object to be drawn is often too large for a sheet of paper. Thus, a scale drawing of the object is drawn to fit the paper. Maps and blueprints are two examples of the use of scale drawings.

On a map, 1 centimetre can represent many kilometres.

Consider the map below. By measuring the length of the road between any two towns, you can calculate the approximate distance using the scale given below. In the diagram, 1 cm represents 28 km.

SCALE:

1 cm ——represents——▶ 28 km

Example To the nearer kilometre, find the approximate distance from Camrose to Strome.

If you measure the distance on the map from Camrose to Strome you will find it to be 1.9 cm. The scale is 1 cm represents 28 km, so, a proportion may now be set up.

$$1 \text{ cm} \xrightarrow{\textit{represents}} 28 \text{ km}$$

$$1.9 \text{ cm} \xrightarrow{\textit{represents}} x \text{ km}$$

$$\frac{1}{1.9} = \frac{28}{x}$$

$$1 \times x = 1.9 \times 28$$

$$x = 53.2$$

$$x = 53 \text{ to the nearer km}$$

Thus, to the nearer km, the distance from Camrose to Strome is 53 km.

EXERCISE 8 Referring to the map on page 18, calculate the approximate distance between the given towns. *Express your answers to the nearer km.*

1. Stettler to Drumheller

EXERCISE 8 (continued)

Calculate the approximate distance, to the nearer km, between the given towns.

2. Hanna to Cereal

3. Lacombe to Mirror

END OF LESSON 26

LESSON RECORD FORM

0902 Math 9
Revised 11/90

FOR STUDENT USE ONLY

Date Lesson Submitted

_____

Time Spent on Lesson

_____

(If label is missing or incorrect)

File Number

_____

Lesson Number _____

FOR A.D.L.C. USE ONLY

Assigned Teacher: _____

Lesson Grading: _____

Additional Grading E/R/P Code: _____

Mark: _____

Graded by: _____

Assignment Code: _____

Date Lesson Received:

_____

Lesson Recorded _____

Student's Questions and Comments

Apply Lesson Label Here

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Please verify that preprinted label is for correct course and lesson.

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Correspondence Teacher

St. Serv. 14-91

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do **not** enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

St. Serv. 9-91

MEASUREMENT

In this lesson you will learn about:

- area
- calculating the area of regular polygons
- different types of prisms, pyramids, cones and cylinders

KEEPING SKILLFUL

Exercise 1

(a) add $x^3 + 5x$ and $3x^2 - 6x + 2$

(b) Subtract $x^2 - 5x$ from $-4x^2 + 2x - 1$

(c) Subtract $-x^3 + 2x$ from $x^3 + 2x^2 - 3x$

(d) Multiply $2x^2 + 5x - 2$ by $5x$

(e) Multiply $x + 3$ and $x - 5$

(f) Divide $15x^2 + 15x - 9x$ by $-3x$

(g) Factor $30x - 18y + 9z$

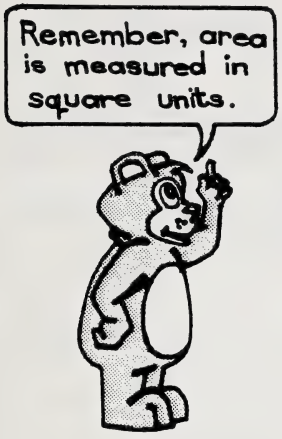
(h) Factor $x^2 - 9$

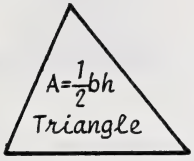
(i) Factor $y^2 - y - 6$

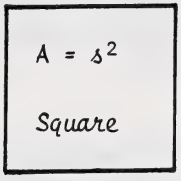
(j) Factor $5y^2 - 80$ completely.

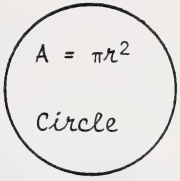
FINDING AREAS

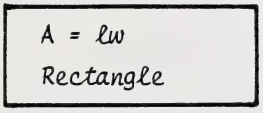
Review the following polygons and their area formulas.

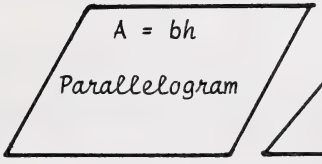


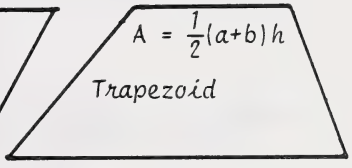












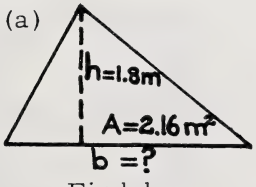
EXERCISE 2 Use the above area formulas to answer the questions below.

1. Calculate the area of each circle. ($\pi = 3.14$)

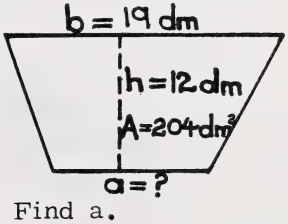
(a)

(b)

2. Calculate the size of the unknown side given the following information.

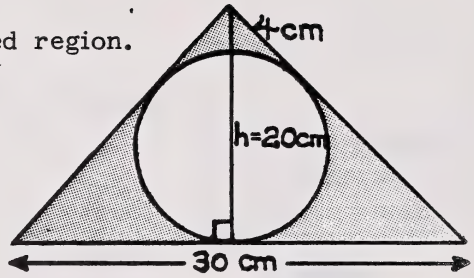
(a) 

Find b.

(b) 

Find a.

Example 5 Calculate the area of the shaded region.



To find the area of the shaded part you will have to find the area of the triangle and then subtract the area of the circle.

Note: The radius of the circle is not given but you can find the diameter.

$$d = 20 - 4 = 16 \text{ cm}$$

$$r = \frac{1}{2}d = 8 \text{ cm}$$

Area of shaded region = Area of triangle - Area of circle.

$$A = (\frac{1}{2}bh) - (\pi r^2)$$

$$= (\frac{1}{2} \times 30 \times 20) - (3.14 \times 8 \times 8)$$

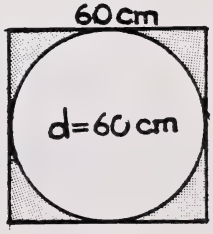
$$= 300 - 200.96$$

$$= 99.04 \text{ cm}^2$$

The area of the shaded part is 99.04 cm^2.

EXERCISE 3 (continued)

3. Calculate the area of the shaded region.



AREA OF SECTORS

A sector is a portion of a complete circle, bounded by two radii and part of the circumference. Informally it may be thought of as a "piece of pie".

Sector

A complete circle measures 360°. A sector will be a fraction of 360°. So, the area of a sector of a circle is:

$$A = \frac{n}{360} \times \pi r^2$$

n represents the degrees of the sector angle.

Example

If a circle has a radius of 10 dm, find the area of a sector whose sector angle measures 60°.

$$A = \frac{n}{360} \times \pi r^2$$

$$A = \frac{\cancel{60}^{1}}{\cancel{360}_{6}} \times 3.14 \times 10 \times 10$$

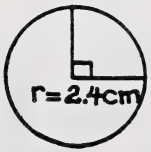
$$A = \frac{1}{6} \times 3.14 \times 10 \times 10$$

$$A = 52.3 \text{ dm}^2$$

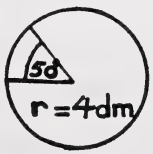
The area of the sector is about 52.3 dm^2.

EXERCISE 4 1. Find the area of each sector. Use $\pi = 3.14$.

(a)



(b)



AREA OF REGULAR POLYGONS

Reference Study page 178 of *Holt Mathematics 3.*

Remember that the area of a triangle is given as $A = \frac{1}{2}bh$. It could also be written as $A = \frac{1}{2}ba$. a and h are the same dimension in each case. They represent height or altitude.

A regular polygon is made up of congruent triangles. The number of sides a regular polygon has indicates the number of congruent triangles. For example, a pentagon has 5 sides or 5 congruent triangles.

The area of a pentagon would be:

$$A = 5 \times \text{area of one triangle}$$

$$A = 5 \times \frac{1}{2}ab$$

If a regular polygon has n sides (or n congruent triangles) then its area will be:

$$A = n \times \frac{1}{2}ab$$

or

$$A = n \times \frac{ab}{2}$$

> $A = \frac{nab}{2}$ when "b" is the base and "a" represents the distance at right angles from the centre of the polygon to the base.

Reference Study the example on page 178 of *Holt Mathematics 3.*

EXERCISE 5 1. State the number of sides each regular polygon would have. Use a dictionary if necessary.

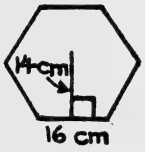
(a) octagon _____ (d) square _____

(b) hexagon _____ (e) heptagon _____

(c) nonagon _____

2. Calculate the area of each regular polygon.

(a)

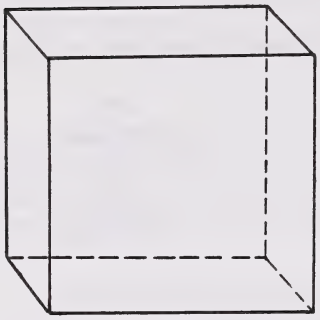


(b)

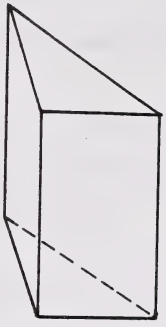
3. A circular picture is placed over a regular hexagonal mirror. What is the area of the mirror that is left uncovered (the shaded part)?

PRISMS

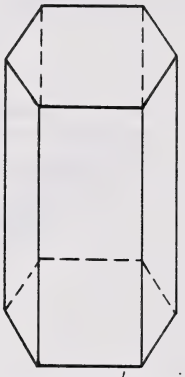
A prism is a solid figure whose ends (bases) are parallel and congruent and whose sides (faces) are polygons. A rectangular solid is another name for a particular kind of prism. Other names of prisms could be triangular prisms, pentagonal prisms, hexagonal prisms and so on, depending on the shape of their bases.



rectangular prism

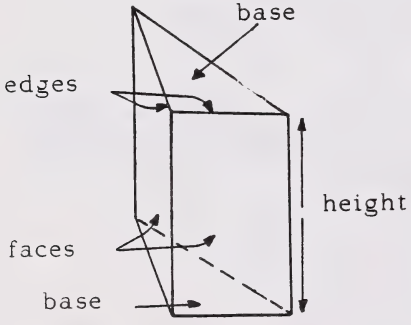


triangular prism



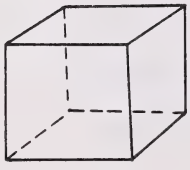
hexagonal prism

The Parts of a Prism



The ends of a prism are its bases. (A prism has two bases.) The sides are called faces or lateral faces. The number of faces varies with the shape of the base. A rectangular prism has four faces, while a hexagonal prism has six faces. The edge is where the base and a face meet or where two faces meet The height of a prism is the length of its lateral face.

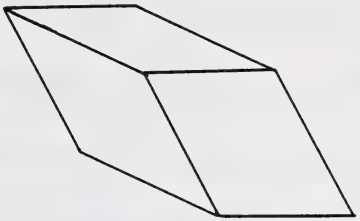
Cube or Square Prism



A cube is a particular kind of prism that has the base the same size as the lateral faces. It is sometimes called a square prism. Its faces and base are squares.

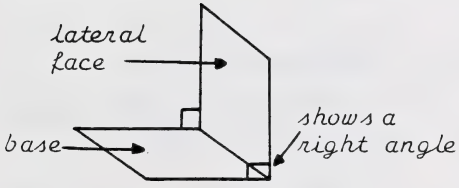
Right Prism

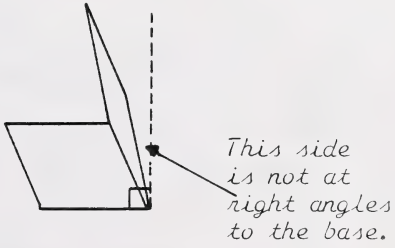
A right prism is a prism whose lateral faces are perpendicular to the base. The diagrams on page 8 are all right prisms.



*This is **not** a right prism*

To be a right prism the lateral faces must be at 90° (right angles) to the base.

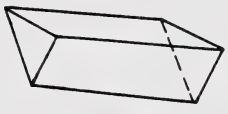




EXERCISE 6 Do the following questions about prisms.

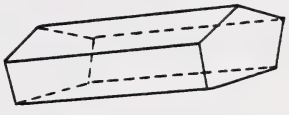
1. Name the following types of prisms according to the shape of their bases.

(a)



_____

(b)

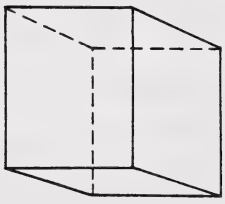


_____

(c)

_____

(d)



_____

EXERCISE 6 (continued)

2. Write either true or false beside the following statements.

(a) _____ The base of a prism can also be the top.

(b) _____ A hexagonal prism has six lateral faces.

(c) _____ A pentagonal prism has five edges.

(d) _____ The face of a prism must be the same shape as the base.

(e) _____ A triangular prism has lateral faces shaped like triangles.

(f) _____ A right prism must have rectangular sides.

(g) _____ A right prism is another name for a rectangular prism.

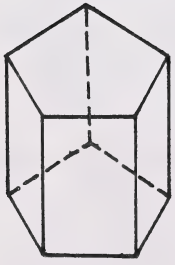
(h) _____ All hexagonal prisms have the same number of edges.

(i) _____ A right regular prism has faces that are regular polygons.

(j) _____ A prism cannot have only two faces.

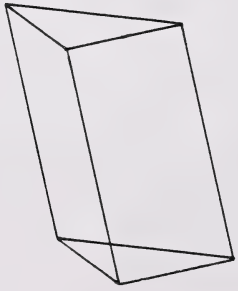
3. Tell whether the following are right prisms or not. Write Yes or No in the blanks.

(a)



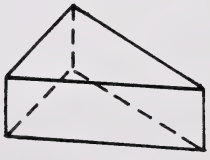
_____

(b)



_____

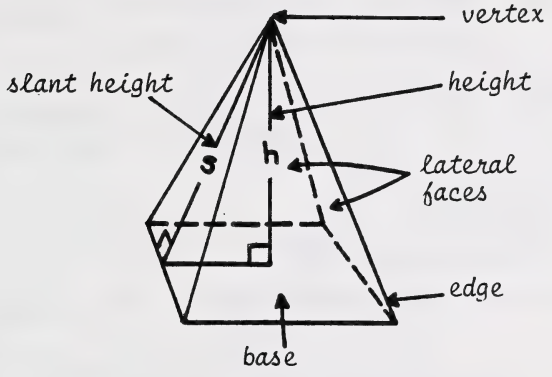
(c)



_____

PYRAMIDS

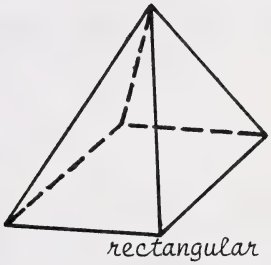
A pyramid is a solid whose base is a polygon and whose lateral faces are triangles that meet at a common vertex.



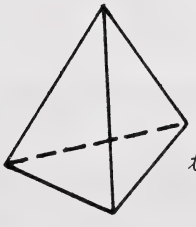
The faces (sides) of a pyramid are triangles. The base (bottom) is a polygon. The edges are where two faces or a face and base meet. The slant height (s) is the length of a line drawn along a face of the pyramid from the vertex perpendicular to the edge of the base. The height (h) is the perpendicular distance from the vertex to the base.

Other examples of pyramids are shown below.

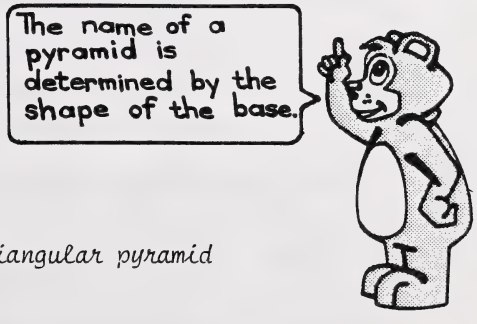
hexagonal pyramid



rectangular pyramid

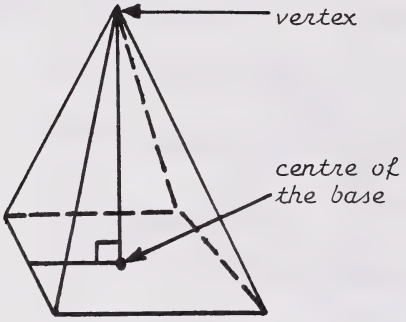


triangular pyramid

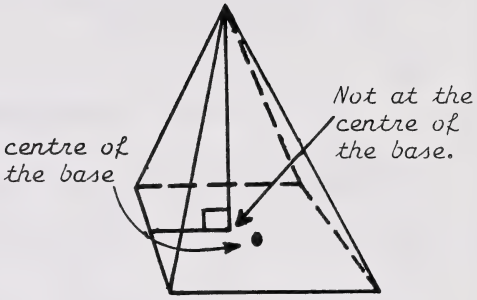


Right Pyramid

A right pyramid is one whose vertex is directly above the centre of the base.



This is a right pyramid.



This is not a right pyramid.

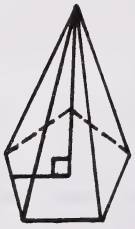
In other words a line drawn through the vertex, at right angles to the base, goes through the centre of the base.

All regular pyramids are right pyramids. A regular pyramid has as its base a regular polygon and its faces are congruent triangles.

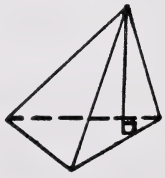
EXERCISE 7 Answer the following questions about pyramids.

1. Which of the following are right pyramids? Explain below why or why not.

(a)



(b)



(a) _____

_____

(b) _____

_____

2. Tell whether the following statements are true or false.

(a) A vertex of a right pyramid is:

_____ always directly above the centre of the base.
_____ where all the faces meet.
_____ is one half the length of the base.
_____ has four edges.

(b) The faces of a regular right pyramid:

_____ are all at right angles to the base.
_____ are congruent polygons.
_____ must always total to four.
_____ are all congruent triangles.

(c) A right octagonal pyramid:

_____ has eight faces.
_____ has more than eight edges.
_____ has eight corners to its base.
_____ always has a vertex that is directly above the centre of the base.

(d) A regular triangular pyramid:

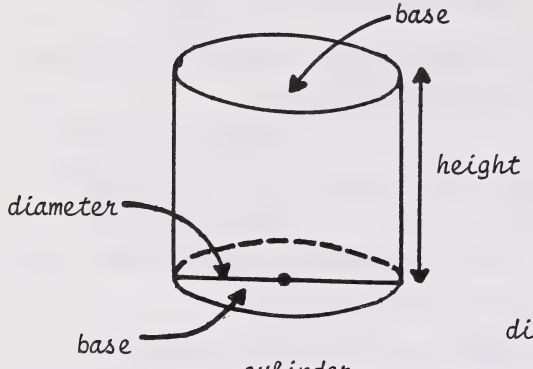
_____ has a base that is an equilateral triangle
_____ has three congruent lateral faces.
_____ has seven edges.
_____ has a slant height.

(e) The slant height of a regular pyramid:

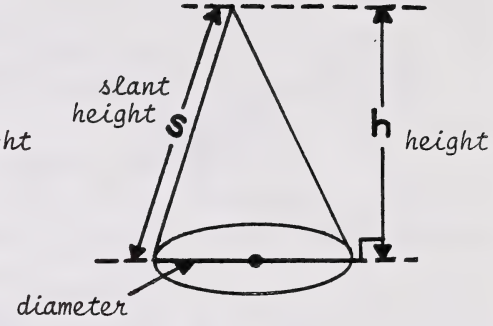
_____ is always perpendicular to the edge of the base.
_____ can be determined from any face.
_____ is measured from the centre of the pyramid.
_____ cannot be equal to the height of the pyramid.

CYLINDERS AND CONES

A cylinder has two circular bases and a lateral surface that is curved. A cone has one circular base and a lateral surface that is curved and that comes to a point.



cylinder



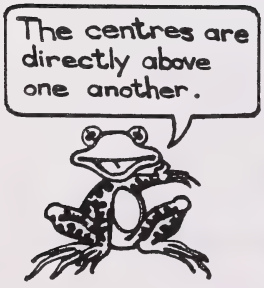
cone

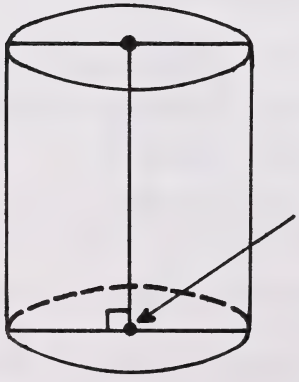
The height (h) (sometimes considered the vertical height) is found by measuring the perpendicular distance from the top to the base.

The slant height (s) is measured from the vertex along the side of the cone to the base.

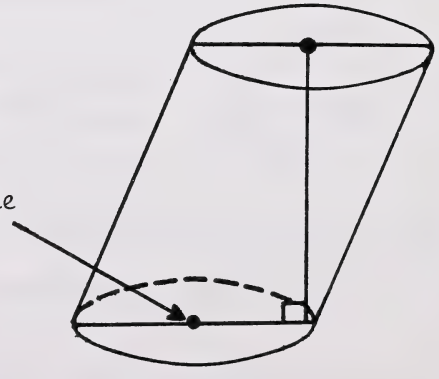
Right Cylinders

A right cylinder has the centre of the top base directly over the centre of the bottom base and its lateral sides at right angles to the bases.





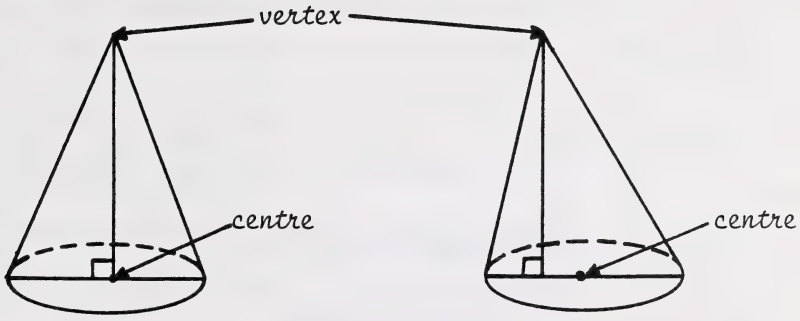
This is a right cylinder.



This is <u>not</u> a right cylinder.

Right Cones

A right cone has its vertex directly above the centre of the base.



This is a right cone. *This is not a right cone.*

Notice that the height of the cylinders and cones are all perpendicular to the base. However, not all of them go through the centre of the base.

EXERCISE 8

Part A

Answer the following questions about cylinders and cones.

1. Write true or false beside the following statements.

_____ The height of a cylinder is always perpendicular to the base.

_____ The slant height of any right cone is the same all around the solid.

_____ The vertex of a cylinder is above the base.

_____ The cone has a round diameter.

_____ The centre of the base of a right cone is directly beneath the vertex.

_____ The diameter of a cone must be less than the height.

_____ A solid figure cannot be a cylinder or a cone unless it has a circular base.

Part B Answer the following questions about prisms, pyramids, cylinders and cones.

1. Complete the table and answer the questions below.

Solid	Number of sides of the base	Number of lateral faces	Number of edges
triangular prism	3	3	9
triangular pyramid			
rectangular prism			
rectangular pyramid			
pentagonal prism			
hexagonal pyramid			

What did you notice about:

(a) the number of sides of the base compared to the number of lateral faces?

_____

_____

(b) the number of edges of the pyramid compared to the number of sides of the base?

_____

_____

(c) the number of edges of the prism compared to the number of sides of the base?

_____

_____

2. (a) What is the least number of faces a pyramid can have? _____

(b) What is the least number of faces a prism can have? _____

(c) A sharpened pencil has what shape? _____

_____

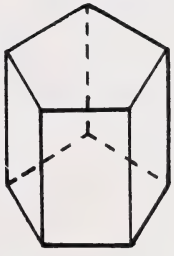
(d) If all the faces of a prism are square, what is the name of the solid? _____

3. Name the following solids.

(a)

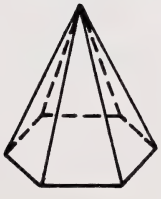
_____

(b)



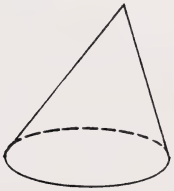
_____

(c)



_____

(d)



_____

4. Match the following right solids with the proper diagram.

right cone _____

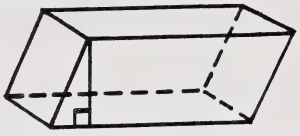
right prism _____

right cylinder _____

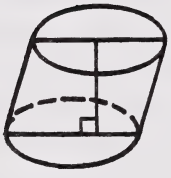
right pyramid _____

(a)

(b)

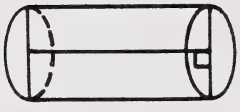


(c)

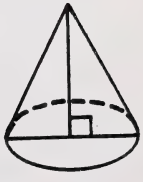


(d)

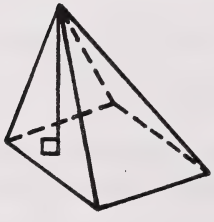
(e)



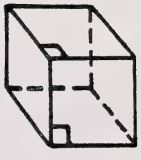
(f)



(g)



(h)



END OF LESSON 27

LESSON RECORD FORM

0902 Math 9
Revised 11/90

FOR STUDENT USE ONLY

Date Lesson Submitted

_____

Time Spent on Lesson

_____

(If label is missing or incorrect)

File Number

_____

Lesson Number _____

FOR A.D.L.C. USE ONLY

Assigned Teacher: _____

Lesson Grading: _____

Additional Grading E/R/P Code: _____

Mark: _____

Graded by: _____

Assignment Code: _____

Date Lesson Received:

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Lesson Recorded _____

Student's Questions and Comments

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Please verify that preprinted label is for correct course and lesson.

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St. Serv. 14-91

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do **not** enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

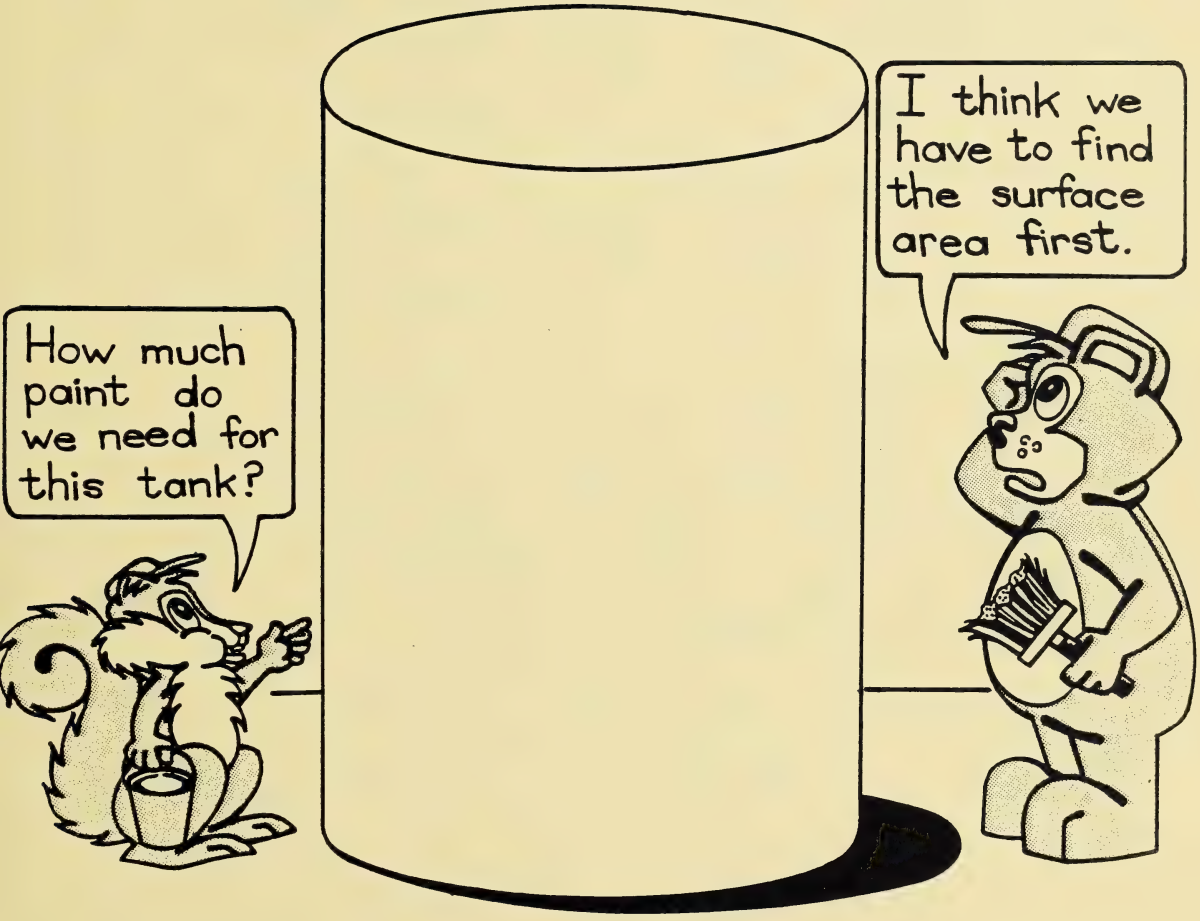
Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

St. Serv. 9-91

GEOMETRY



In this lesson you will learn about:

- constructing models of prisms, pyramids and other regular polyhedra
- calculating the surface area of prisms, pyramids, and cylinders

KEEPING SKILLFUL

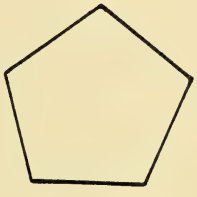
EXERCISE 1 Match Part One with Part Two.

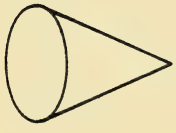
Part One

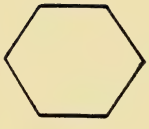
f	rectangular prism	____	rectangular pyramid
____	isosceles triangle	____	scalene triangle
____	pentagon	____	hexagon
____	cone	____	equilateral triangle
____	triangular pyramid	____	trapezoid
____	octagon		

Part Two

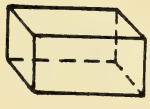
(a)

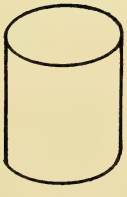
(b) 

(c) 

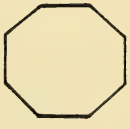
(d) 

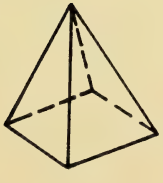
(e)

(f) 

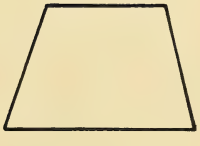
(g) 

(h)

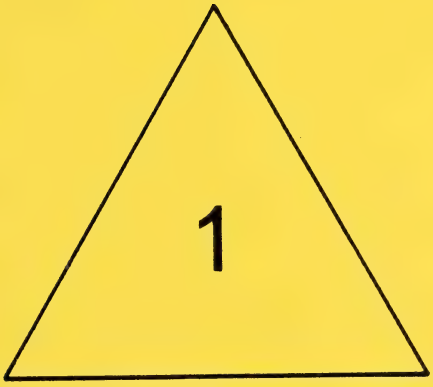
(i) 

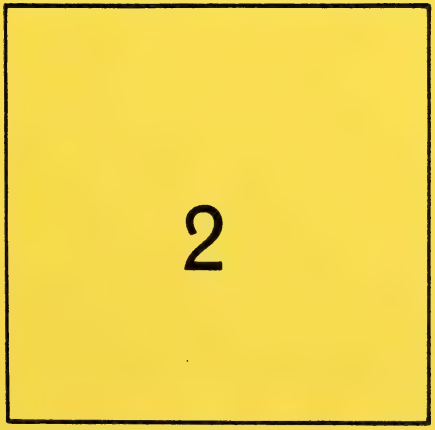
(j) 

(k)

(l) 

Patterns to Use





CONSTRUCTING MODELS

You have studied different types of solids in Lesson 27. In Lesson 27 there were drawings of some prisms, pyramids, cylinders and cones. It is difficult to show the accurate shape of a solid on a flat piece of paper. To help you see what different solids look like, you may wish to construct some models.

Follow the directions below.

1. Cut out the shapes on page 4 of this lesson. Trace these onto stiff paper to form the patterns shown on pages 3, 5, 6 and 7. (Coloured paper makes attractive models.)

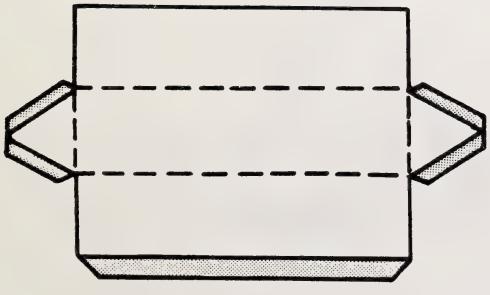
2. The shaded areas in the patterns are tabs. These hold the model together. Draw the tabs where indicated on the pattern.

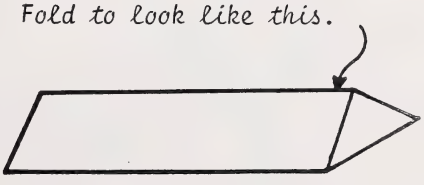
3. When you have finished your drawing, cut the shape along the solid lines. Fold along the dotted lines.

4. Form the models and fasten the tabs using glue or tape.

TRIANGULAR PRISM

You will need a piece of paper 20 cm long by 20 cm wide. Use pattern ⚠ for the triangles. Each rectangle should be 6.5 cm by 10 cm.

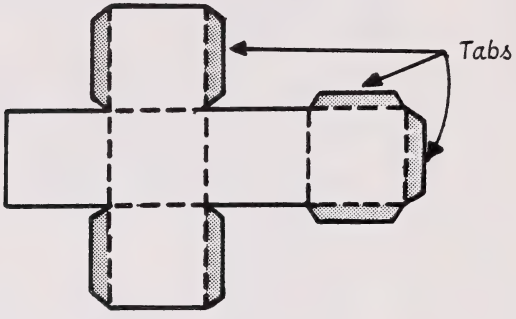


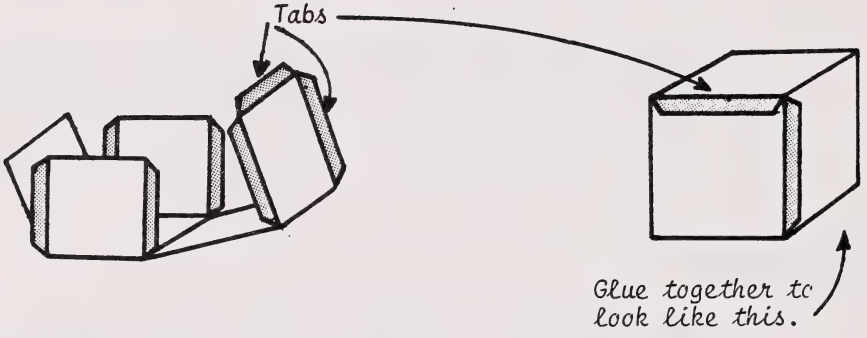


Label this model a triangular prism.

CUBE

You will need a piece of paper 30 cm long and 20 cm wide. Use pattern 2 to draw the shape below.



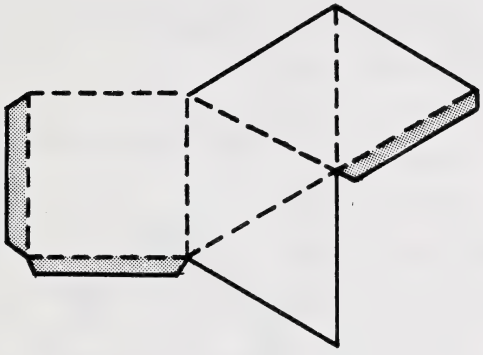


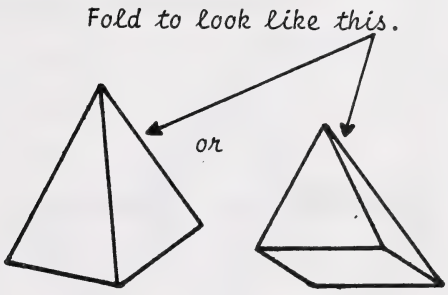
Label this model a cube. Another name for a cube is a hexahedron. It has six faces in the shape of squares.

If you wish to show your teacher one or more of your models, send them with this lesson. Put your name on each model and leave them flat without the tabs glued.

SQUARE PYRAMID

You need a sheet of paper 20 cm by 13 cm. Draw the shape below using patterns ① and ② .



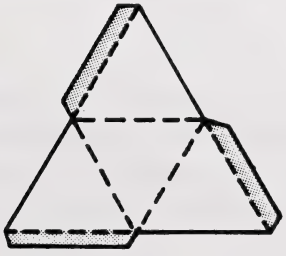


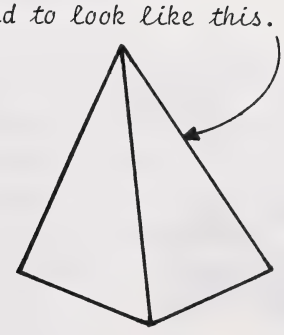
Label this model a square pyramid.

TRIANGULAR PYRAMID

Another name used for a triangular pyramid is a tetrahedron. It has four faces in the shape of equilateral triangles.

You need a sheet of paper 13 cm wide and 13 cm long. Draw the shape below using pattern ① .



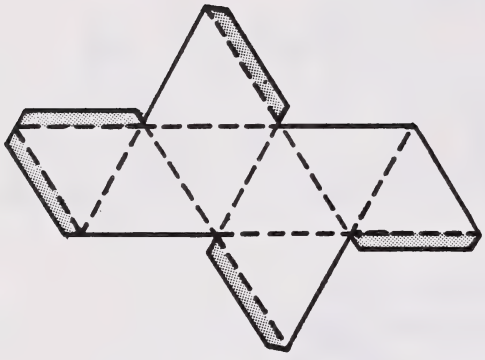


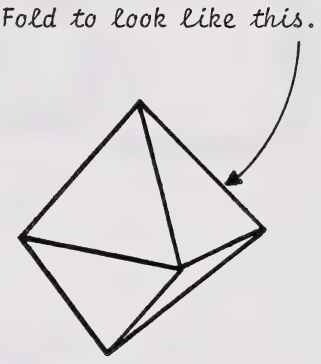
Label this model a triangular prism or tetrahedron.

OCTAHEDRON

An octahedron is a solid having 8 faces. A regular octahedron is one whose faces are all congruent equilateral triangles.

You need a sheet of paper 20 cm by 30 cm. Draw the shape below using pattern ⚠ .



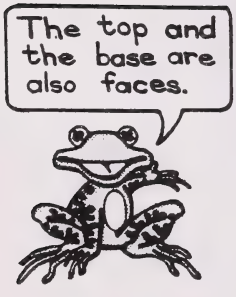


Label this model an octahedron.

FINDING THE SURFACE AREA OF PRISMS

You may want to, first of all, review the different kinds of prisms you studied in Lesson 27.

The surface area of a prism is found by adding the area of each face.



Surface Area = Sum of the Areas of the Faces

To find the surface area you will have to know the length, width and height of your prism.

Rectangular Solid

A rectangular solid has six faces. To find the surface area, the area of each face must be determined. However, remember that in a rectangular solid the top and base will have the same dimensions and, the faces opposite each other will have the same dimensions.

Area of top = $\ell \times w = \ell w$

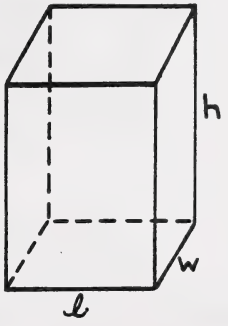
Area of base = $\ell \times w = \ell w$

Area of front = $\ell \times h = \ell h$

Area of back = $\ell \times h = \ell h$

Area of left side = $w \times h = wh$

Area of right side = $w \times h = wh$



To find the surface area, you can add the area of each face or use the following formula.

$$\text{Surface Area} = \ell w + \ell w + \ell h + \ell h + wh + wh$$
$$= 2\ell w + 2\ell h + 2wh$$

If SA stands for surface area, the formula is:

Formula

$$SA = 2\ell w + 2\ell h + 2wh$$

You may use the above formula for finding the surface area of any rectangular prism.

Example

Find the surface area of a rectangular prism whose dimensions are: width 6 cm, length 8 cm and height 5 cm.

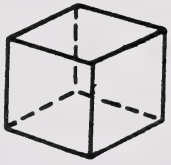
$$SA = 2\ell w + 2\ell h + 2wh$$
$$= 2(8 \times 6) + 2(8 \times 5) + 2(6 \times 5)$$
$$= 2(48) + 2(40) + 2(30)$$
$$= 96 + 80 + 60$$
$$= 236 \text{ cm}^2$$

The surface area of this prism is 236 cm^2.

Cube

A cube is a particular kind of rectangular prism; edges are all of equal length. Since each edge is the same size, each face will have the same area.

Since the length, width, and height are the same, "s" is used to represent them. Then, the area of one side is s^2.



There are 6 faces so the formula for the area of a cube would be:

Formula

$$SA = 6s^2$$

Example

A cube has a side that is 1.5 mm long. What is its surface area?

$$\begin{aligned} SA &= 6s^2 \\ &= 6(1.5)^2 \\ &= 6(1.5 \times 1.5) \\ &= 6(2.25) \\ &= 13.5 \text{ mm}^2 \end{aligned}$$

The surface area of the cube is 13.5 mm^2.

Right Regular Prisms

The measure of the total surface area of a regular prism is found by adding the area of the base and top to the area of the lateral faces.

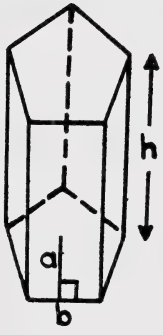
A general formula for surface area may be derived. The formula for the area of any regular polygon is $A = \frac{nab}{2}$. [See Lesson 7 or page 178 of *Holt Mathematics 3.*] In the formula $A = \frac{nab}{2}$, "n" represents the number of sides of the polygon, "a" is the distance from the centre to the edge (at right angles) and "b" is the length of one side of the base.

Reference Study page 189, column one of *Holt Mathematics 3.*

A general formula for the surface area of a regular prism can be developed by studying any regular prism. We will use a pentagonal prism as a model.

A pentagon is a polygon that has 5 sides (n = 5).

Now study the pentagonal prism below.



Area of the base is $\frac{nab}{2}$.

n is the number of sides of the base.

Area of the top is $\frac{nab}{2}$.

Area of each side is bh.

Area of n sides is nbh.

So, the surface area of any regular prism is:

$$SA = \frac{nab}{2} + \frac{nab}{2} + nbh$$

$$= nab + nbh$$

$$= nb(a + h)$$

$\frac{nab}{2}$ is the same as $\frac{1}{2}$ nab

So, $\frac{1}{2}$ nab + $\frac{1}{2}$ nab = nab.

Formula The general formula for any regular prism that is <u>not a rectangular solid</u> is:

$$SA = nb(a + h)$$

Example

A regular octagonal prism has a height of 11 dm. The perpendicular distance across the base going through the centre is 8 dm and the measure of one side of the base is 6 dm.

$n = 8$ ← *Octagonal prisms have 8 sides.*

$a = 8\text{dm} \div 2 = 4\text{dm}$ ← *"a" is the distance from the centre of the base to the edge.*

$b = 6$ dm

$h = 11$ dm

$$SA = nb(a + h)$$
$$= 8 \times 6(4 + 11)$$
$$= 48(15)$$
$$= 720 \text{ dm}^2$$

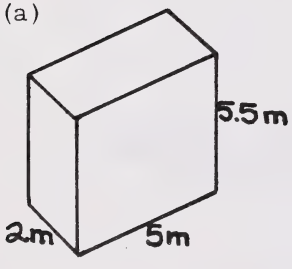
The surface area of the octagonal prism is 720 dm^2.

EXERCISE 2

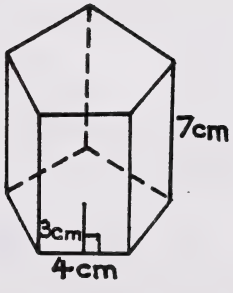
Use the formulas already discussed to answer the following questions.

1. Find the surface area.

(a)



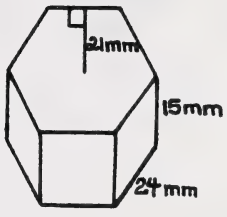
(b)



2. A block of ice is in the shape of a rectangular prism. Its width is 15 dm, length 20 dm and height 17 dm. What would the surface area of this block of ice be?

3. A cube of sugar measures 1.2 cm on each side. What is its surface area?

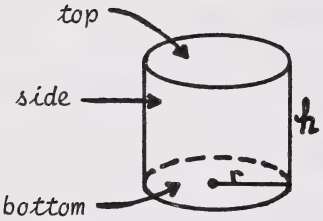
4. Find the surface area of the following right prism.

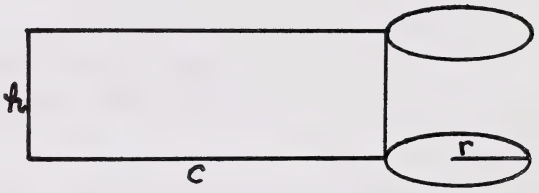


FINDING THE SURFACE AREA OF CYLINDERS

Reference Study 189, column two, of *Holt Mathematics 3.*

To find the surface area of a cylinder you will have to find the area of the base, the top and the round side. If the round side is rolled out it forms a rectangle. See the diagram below.





Area of the base = πr^2

Area of the top = πr^2

Area of the side = $h \times C$

= $h \times 2\pi r$

= $2\pi rh$

The length is really the circumference (C) of the circle. C = 2πr

Surface area = $\pi r^2 + \pi r^2 + 2\pi rh$

= $2\pi r^2 + 2\pi rh$

= $2\pi r(r + h)$

Formula The formula for the surface area of a cylinder is:

$$SA = 2\pi r(r + h)$$

Example

A cylindrical tank has a radius of 12 m and a height of 20 m. What is its surface area? (Use $\pi = 3.14$.)

$$SA = 2\pi r(r + h)$$

$$= 2 \times 3.14 \times 12 \ (12 + 20)$$

$$= 6.28 \times 12 \ (32)$$

$$= 75.36 \times 32$$

$$= 2411.52 \text{ m}^2$$

The surface area of the tank would be 2411.52 m^2 .

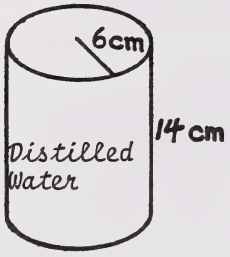
EXERCISE 3

Do the following questions dealing with the surface area of cylinders.

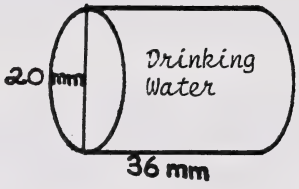
1. A holding tank in the shape of a cylinder has a diameter of 4 m and a height of 100 m. What would its surface area be? (Hint: first find the radius.)

2. Find the surface area of each cylinder.

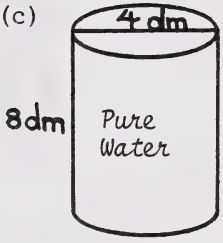
(a)



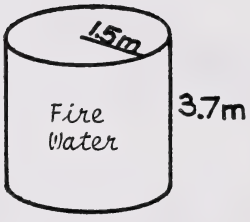
(b)



(c)



(d)



SURFACE AREA OF PYRAMIDS

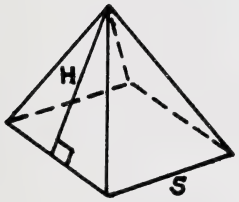
The formula for the surface area of a pyramid will depend on the shape of the base of the pyramid. To find the surface area of any pyramid, you will have to find the area of the base and add the area of the lateral faces (sides).

A square-based pyramid will have four lateral faces, while a triangular pyramid has three lateral faces. A pentagonal pyramid has five lateral faces. To find the area of each face, the slant height (your textbook uses the symbol H for slant height) will have to be known.

Reference

Study page 191, column one, of *Holt Mathematics 3.*

Surface Area = area of base + area of lateral faces



For a square-based pyramid:

Surface Area $= s^2 + 4(\frac{1}{2} Hs)$ ← 4 faces.

$= s^2 + 2Hs$ ← "s" is the length of each side.

or $= 2Hs + s^2$

Formula

$$SA = 2Hs + s^2$$

This formula can only be used for square-based pyramids.

Example If a certain square-based pyramid has a slant height of 16 m and if the length of one side of the base is 12 m, what is the surface area?

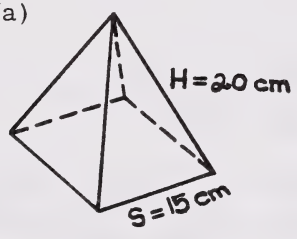
$$\begin{aligned} SA &= 2Hs + s^2 \\ &= (2 \times 16 \times 12) + (12)^2 \\ &= 384 + 144 \\ &= 528 \text{ m}^2 \end{aligned}$$

The surface area would be 528 m^2.

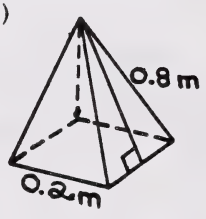
EXERCISE 4 The following questions deal only with square-based pyramids.

1. Find the surface area of each pyramid.

(a)



(b)



2. The Great Pyramid of Cheops is in Egypt. Its slant height is 186.4 m and the length of one side of its square base is 230.3 m. Find the surface area.

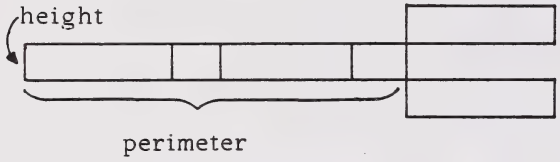
Summary of Formulae For Surface Area		
1.	Rectangular Solid	$SA = 2lw + 2lh + 2wh$
2.	Cube	$SA = 6s^2$
3.	Regular prism	$SA = nb\ (a + h)$
4.	Cylinder	$SA = 2\pi r\ (r + h)$
5.	Square based pyramid	$SA = 2Hs + s^2$

General Formulae for Prisms and Cylinders

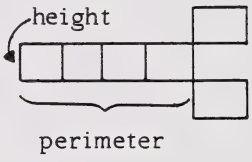
A general formula for the surface area of all prisms and cylinders can also be developed.

If you "unwrap" each of the models, you will notice the following:

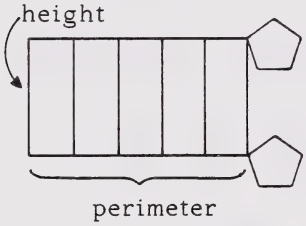
Rectangular Solid:



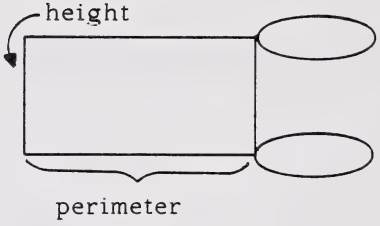
Cube:



Regular Polygon:



Cylinder:



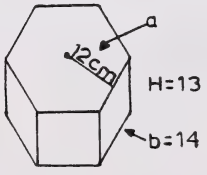
To find the surface area of any prism or cylinder you can, therefore, add the area of the bases (B) to the area of rectangle formed by the perimeter (P) and the height (H).

$$SA = 2(B) + PH.$$

Note: This formula does not work for pyramids.

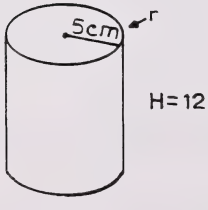
Example: Find the surface area of the following.

(a)



$$\begin{aligned} SA &= 2B + PH \\ &= 2\left(\frac{nab}{2}\right) + 6(b)\,H \\ &= 6 \times 12 \times 14 + 6 \times 14 \times 13 \\ &= 1008 + 1092 \\ &= 2100 \text{ cm}^2 \end{aligned}$$

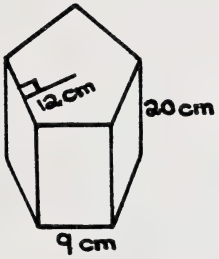
(b)



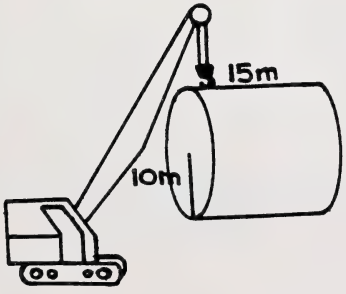
$$\begin{aligned} SA &= 2B + PH \\ &= 2(\pi r^2) + (\pi d)\,(12) \\ &= 2(3.14)(5)(5) + (3.14)(10)(\\ &= 157 + 376.8 \\ &= 533.8 \text{ cm}^2 \end{aligned}$$

EXERCISE 5

1. A base of a lamp is in the shape of a pentagonal prism. How much surface area of glass is needed to cover all the faces. Use SA = 2B + PH.



2. Jason wants to know how much paint to buy to paint a storage tank. If one litre of paint covers 15.7 m^2 , how many litres must Jason buy to cover the following tank. Use SA = 2B + PH and π = 3.14



End of Lesson 28

LESSON RECORD FORM

0902 Math 9
Revised 11/90

FOR STUDENT USE ONLY

Date Lesson Submitted

_____

Time Spent on Lesson

_____

(If label is missing or incorrect)

File Number

_____

Lesson Number _____

Student's Questions and Comments

Apply Lesson Label Here

Name

Address

Postal Code

Please verify that preprinted label is for correct course and lesson.

FOR A.D.L.C. USE ONLY

Assigned Teacher: _____

Lesson Grading: _____

Additional Grading E/R/P Code: _____

Mark: _____

Graded by: _____

Assignment Code: _____

Date Lesson Received:

_____

Lesson Recorded _____

Teacher's Comments:

_____

Correspondence Teacher

St. Serv. 14-91

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do **not** enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for **correction regularly. Avoid sending more than two or three lessons in one subject at the same time.**

St. Serv. 9-91

GEOMETRY

In this lesson you will learn about finding the volume of:

- prisms
- cylinders
- pyramids
- cones

KEEPING SKILLFUL

Exercise 1 Factor the following polynomials completely.

1. $24abc - 30a^2bc$ = _____
2. $5y^3 - 25y$ = _____
3. $x^2 - 144$ = _____
4. $9b^2 - 25$ = _____
5. $x^2 - 10x + 25$ = _____
6. $x^2 - 9x + 14$ = _____
7. $b^2 + 14b + 40$ = _____
8. $x^2 - x - 30$ = _____
9. $x^3 - x^2 + x$ = _____
10. $3x^2 - 12$ = _____

VOLUME OF PRISMS

You may want to review perimeter and area in Lesson 27 before attempting this section.

Reference

Read column one on page 181 of *Holt Mathematics 3*.

The volume of any prism is found by multiplying the area of its base by its height.

In your textbook "B" stands for the area of the base and "h" represents the height. So, the formula for the volume of a prism is:

$$V = Bh$$

The height (h) of a prism will usually be given to you. However, to find the area of the base (B) a few things must be considered.

1. You must recognize the shape of the base.
2. You must know the formula for the area of that shape.

Rectangular Prism

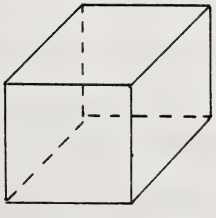
The shape of the base of a rectangular prism is a rectangle.

The formula for the area of a rectangle is $A = \ell w$.

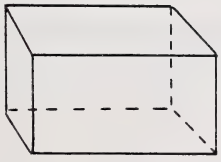
So, the formula for the volume of a rectangular prism can be changed from $V = Bh$ to:

$$V = \ell wh$$

The area of the base, B, is equal to ℓw.

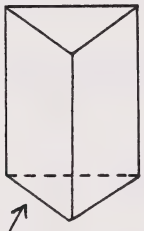


square base



rectangular base

Triangular Prism



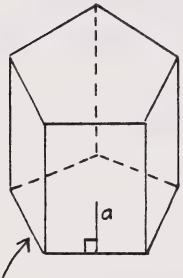
Triangular Base

A triangular prism has a triangle as its base.

The formula for the area of a triangle is: $A = \frac{1}{2}ab$.

To find B, in $V = Bh$, you may substitute $\frac{1}{2}ab$ for B. The formula for the volume of a triangular prism may now be changed from $V = Bh$ to: $V = (\frac{1}{2}ab)h$.

Pentagonal Prism

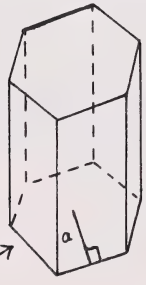


The base is a pentagon. (n = 5)

A pentagonal prism has a base that is in the shape of a pentagon.

The formula for the area of a regular pentagon is $A = \frac{nab}{2}$, where n is the number of sides, b is the length of one side, and a is the perpendicular distance from the centre called the apothem. (See page 178 of your textbook.) The volume of a pentagonal prism may now be changed from $V = Bh$ to: $V = (\frac{nab}{2})h$.

Hexagonal Prism



The base is a hexagon. (n = 6)

A hexagonal prism has a hexagon as the shape of its base.

The formula for the area of a regular hexagon is also $A = \frac{nab}{2}$, where n is the number of sides, b is the length of one side, and a the perpendicular distance from the centre (the apothem).

The volume of a hexagonal prism may now be changed from $V = Bh$ to: $V = (\frac{nab}{2})h$.

Other Prisms

This lesson will only deal with bases that have a regular shape. Therefore, the general formula for the area of a regular polygon may be used, which is $A = \frac{nab}{2}$.

The formula for the volume of any regular prism can then be changed from $V = Bh$ to:

$$V = \left(\frac{nab}{2}\right)h$$

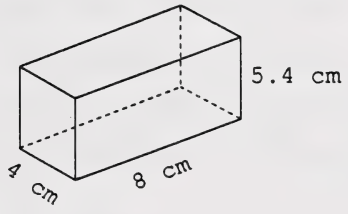
n is the number of sides of the base,

b is the length of one side and,

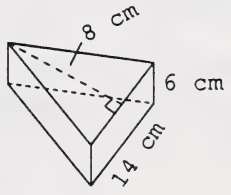
a is the perpendicular distance from the centre (the apothem).

SELF-CORRECTING EXERCISE A

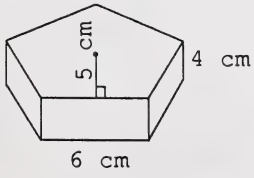
1. Calculate the volume of the rectangular prism.



2. Calculate the volume of the triangular prism.



3. Calculate the volume of the right regular prism.



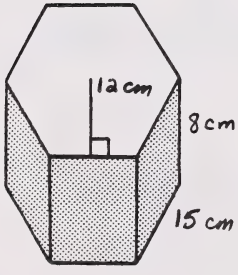
Check your answers with those on page 21 of this lesson. Correct any erors in your work.

EXERCISE 2 Do the following questions about the volume of prisms.

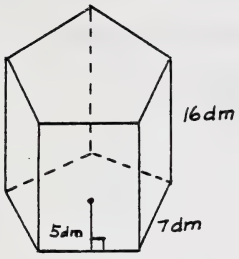
1. A certain classroom is 11 m long, 6.5 m wide and 4.3 m high. How many cubic metres of air space does the room contain?

2. Find the volume of a cube whose sides measure 17 mm.

3. A jewelry case is in the shape of a regular hexagonal prism. Find its volume.



4. Find the volume of the following regular prism.

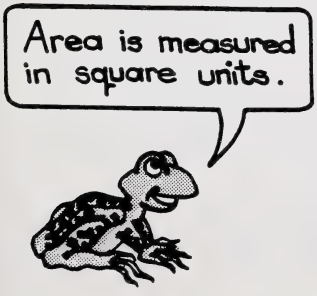


FINDING SURFACE AREA OR HEIGHT WHEN THE VOLUME IS KNOWN

The problems you have encountered so far involved finding the volume. However, sometimes you may know the volume and must find the height or the area of the base. Study the following examples.

Example 1 If the volume of a rectangular prism is 2240 mm^3, and if its height is 40 mm, find the area of the base.

$V = 2240$ $B = ?$ $h = 40$



$V = Bh$

$2240 = B \times 40$

$2240 \div 40 = B$

$56 = B$

$$40\overline{)2240} = 56$$

$200\times$

240

240

00

The area of the base would be 56 mm^2.

Example 2 If the volume of a regular hexagonal prism is 504 m^3, the apothem is 3 m, the length of each side 4 m, find the height.

Use $V = Bh$, but remember that B in this case becomes $\frac{nab}{2}$. Thus, the changed formula is, $V = \frac{nab}{2} \times h$.

$V = 504, \quad n = 6, \quad a = 3, \quad b = 4, \quad h = ?$

$$504 = \frac{\overset{3}{\cancel{6}} \times 3 \times \overset{2}{\cancel{4}}}{\underset{1}{\cancel{2}}} \times h$$

$$504 = 3 \times 3 \times 4 \times h$$

$$504 = 36h$$

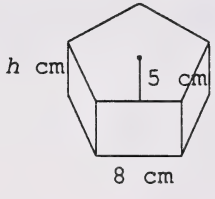
$$504 \div 36 = h$$

$$14 = h$$

The height of this hexagonal prism is 14 m.

SELF-CORRECTING EXERCISE B

1. Calculate the height of the prism if the volume is 900 cm^3.



Check your answer with the one on page 21 of this lesson. Correct any errors in your work.

EXERCISE 3 Do the following questions.

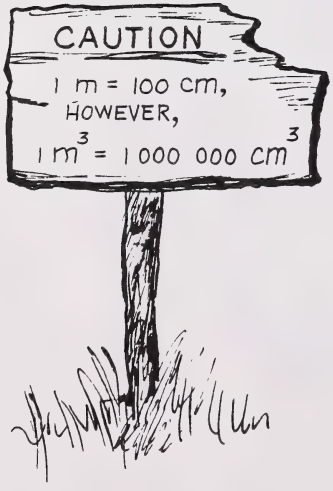
1. Find the height of a rectangular prism that has a volume of 43.056 cm^3 when the length is 4.8 cm and the width is 3.9 cm.

2. A rod is in the shape of a pentagonal prism. If its volume is 112.5 cm^3 and its height is 25 cm, what would the area of its base be?

EXERCISE 3 (continued)

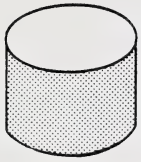
3. The base of a steel tank that is in the shape of a rectangular prism has a length of 5.4 m and a width of 2.3 m. If its volume is 37.26 m^3, what would its height be?

4. A concrete pad is being poured to form the bottom of a statue. This pad is in the shape of a hexagonal prism and can only have a height of 50 cm. The apothem is 200 cm. If the maximum amount of concrete allowed is 9 m^3 find the length of one side of the pad.



VOLUME OF CYLINDERS

Reference Study the top of column one on page 183 of *Holt Mathematics 3.*



The formula for the volume of a cylinder is given as:

$V = Bh$ - where B is the area of the base.
- and h is the height of the cylinder.

Every cylinder has a circular base.

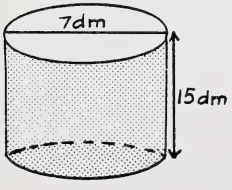
Since the base is a circle you must know the formula for the area of a circle. Remember from Lesson 7 that the formula for the area of a circle is $A = \pi r^2$.

Now, replace B in the basic formula with πr^2. The formula for the volume of a cylinder becomes:

$$V = \pi r^2 h$$

For all calculations use $\pi = 3.14$.

Example 1 Find the volume of the following cylinder.



$V = ?, \quad \pi = 3.14, \quad r = 3.5 \text{ dm}, \quad h = 15 \text{ dm}$

$V = 3.14 \times (3.5)^2 \times 15$

$V = 3.14 \times 12.25 \times 15$

$V = 576.975 \text{ dm}^3$

The volume is 576.975 dm^3.

Example 2 Find the radius of a cylinder whose volume is 62.8 m^3 and whose height is 5 m.

$V = 62.8, \quad \pi = 3.14, \quad r = ?, \quad h = 5$

$$V = \pi r^2 h$$

$$62.8 = 3.14 \times r^2 \times 5$$

$$62.8 = 15.7 \times r^2$$

$$62.8 \div 15.7 = r^2$$

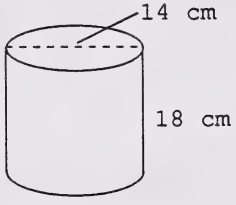
$$4 = r^2$$

$$\sqrt{4} = r$$

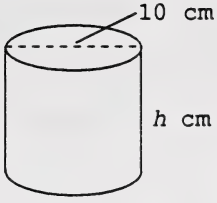
$$2 = r$$

The radius of this cylinder is 2 m.

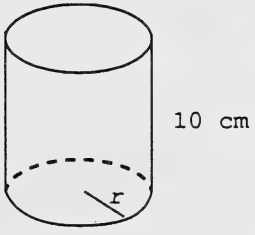
SELF-CORRECTING EXERCISE C

1. 

Find the volume of the cylinder. Use $\pi = 3.14$.

2. 

Find the height of the cylinder if the volume is 600 cm^3 . Use $\pi = 3.14$.

3. 

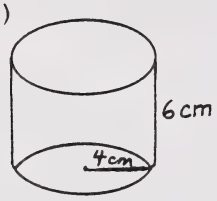
Find the radius of the cylinder if the volume is 785 cm^3 . Use $\pi = 3.14$.

Check your answers with those on page 22 of this lesson. Correct any errors in your work.

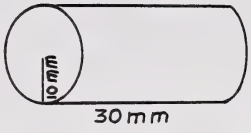
EXERCISE 4 Do the following questions.

1. Find the volume of the cylinders below.

(a)



(b)



2. The volume of a cylinder is 351.68 cm^3 and its radius is 4 cm. Find the height.

3. A can of dried prunes has a volume of 785 cm^3 and a height of 10 cm. Find its radius.

VOLUME OF PYRAMIDS AND CONES

A regular pyramid has one-third the volume of a prism with the same base. A regular cone has one-third the volume of a cylinder with the same base. Thus, the basic formula for the volume of a pyramid or a cone would be:

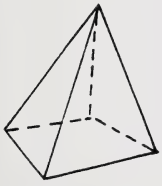
$$V = \frac{1}{3} Bh$$

Reference

Refer to page 185 of *Holt Mathematics 3.*

Rectangular Pyramid

A rectangular pyramid has a base that is in the shape of a rectangle. So, the formula for the volume becomes:

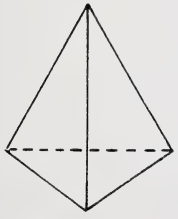


$$V = \frac{1}{3} \ell wh$$

The area of the base, B, is equal to ℓw.

Triangular Pyramid

A triangular pyramid has a base that is in the shape of a triangle. So, the formula for its volume becomes:



$$V = \frac{1}{3} \left(\frac{1}{2} ab \right) h$$

$$V = \frac{1}{6} abh$$

The area of the base, B, is equal to $\frac{1}{2} ab$.

Other Pyramids

For any other regular pyramid, use the formula for the volume of the corresponding prism (given on page 5 of this lesson) and take one-third of it. Thus, the formula becomes:

$$V = \frac{1}{3} \left(\frac{nab}{2} \right) h$$

$$V = \frac{nab}{6} h$$

The area of the base, B, is equal to $\frac{nab}{2}$.

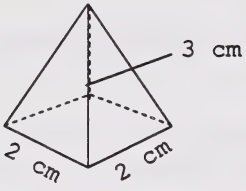
Cone

The cone has a base that is a circle. So, the formula for the volume of a cone becomes:

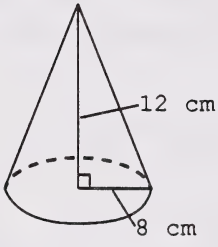
$$V = \frac{1}{3}\pi r^2 h$$

The area of the base, B, is equal to πr^2.

SELF-CORRECTING EXERCISE D

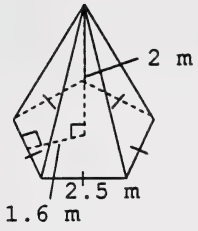
1. 

Calculate the volume of the pyramid.

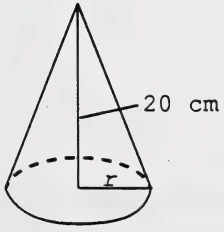
2. 

Calculate the volume of the cone. Use $\pi = 3.14$.

SELF-CORRECTING EXERCISE D (continued)

3. 

Find the volume of the pentagonal pyramid.

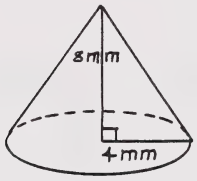
4. 

Find the radius of the cone if the volume is 753.6 cm^3.
Use $\pi = 3.14$.

Hint: First, find r^2. Round off the value of r^2 to the nearer whole number. Then, to find r, take the square root of the value of r^2.

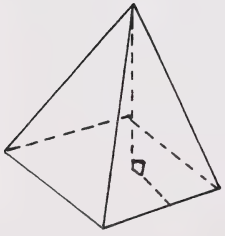
Check your answers with those on page 22 and 23 of this lesson. Correct any errors in your work.

EXERCISE 5 1. Find the volume of the cone below.



2. Find the volume of the pyramid below.

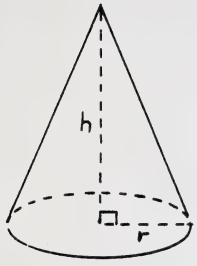
h = 9 cm, ℓ = 6 cm, w = 6 cm



EXERCISE 5 (continued)

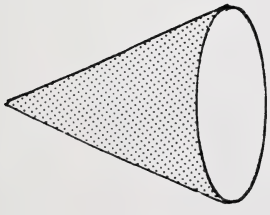
3. Find the height of the cone below.

$V = 94.2\ m^3$, $r = 1.5\ m$



4. Find the radius of the cone below.

$V = 615.44\ cm^3$, $h = 12\ cm$



EXERCISE 6 Applications. Show all the steps to problem solving (variable, equation, solution and computation, and the statement).

1. A well is dug on an acreage to a depth of 200 m. If the radius of the hole is 0.2 m how much earth was removed from the hole?

2. Mr. Goath needs to order cement to put on his driveway that is 5.8 m by 18.5 m. The cement will be poured to a thickness of 10 cm. If the cost of cement is $\$35/m^3$, how much will Mr. Goath spend?

KEY TO SELF-CORRECTING EXERCISES IN LESSON 29

SELF-CORRECTING EXERCISE A, Page 5.

1. This is a rectangular prism. Use the formula $V = \ell wh$

$V = \ell wh$

$= 4 \times 8 \times 5.4$

$= 172.8 \text{ cm}^3$

2. This is a triangular prism. Use the formula $V = Bh$, where $B = \frac{1}{2}ab$

$a = 8$ cm $\quad b = 14$ cm $\quad h = 6$ cm

$V = \left(\frac{1}{2}ab\right)h$

$V = \frac{1}{\cancel{2}} \times \overset{4}{\cancel{8}} \times 14 \times 6$

$V = 336 \text{ cm}^3$

3. This is a pentagonal prism, so use the formula $V = Bh$, where $B = \frac{nab}{2}$

$n = 5 \quad a = 5$ cm $\quad b = 6$ cm $\quad h = 4$ cm

$V = \frac{nab}{2} h$

$= \frac{5 \times 5 \times \overset{3}{\cancel{6}}}{\cancel{2}} \times 4$

$= 300 \text{ cm}^3$

SELF-CORRECTING EXERCISE B, Page 8.

1. $V = 900 \text{ cm}^3 \quad n = 5 \quad a = 5$ cm $\quad b = 8$ cm $\quad h = ?$

Use $V = Bh$

Where $B = \frac{nab}{2}$ $\qquad V = \frac{nab}{2} h$

$900 = \frac{5 \times 5 \times \overset{4}{\cancel{8}}}{\cancel{2}} \times h$

$900 = 100\,h$

$9 = h$ $\qquad$ The height is 9 cm.

SELF-CORRECTING EXERCISE C, pages 12 and 13.

1. This is a cylinder. Use the formula $V = \pi r^2 h$
   $\pi = 3.14$ $r = 7$ cm $h = 18$ cm

$$V = \pi r^2 h$$
$$= 3.14 \times 7 \times 7 \times 18$$
$$= 2769.48 \text{ cm}^3$$

2. $V = 600 \text{ cm}^3$; $r = 5$ cm; $\pi = 3.14$; $h = ?$

$$V = \pi r^2 h$$
$$600 = 3.14 \times 5 \times 5 \times h$$
$$600 = 78.5\ h$$
$$\frac{600}{78.5} = h$$
$$7.64 = h$$

The height is 7.64 cm.

3. $V = 785 \text{ cm}^3$ $r = ?$ $\pi = 3,14$ $h = 10$ cm

$$V = \pi r^2 h$$
$$785 = 3.14 \times r^2 \times 10$$
$$785 = 31.4\ r^2$$
$$\frac{788}{31.4} = r^2$$
$$25 = r^2$$
$$5 = r$$

The radius is 5 cm.

SELF-CORRECTING EXERCISE D, pages 16 and 17.

1. $V = ?$ $\ell = 2$ m $w = 2$ m $h = 3$ m

Use $V = \frac{1}{3} Bh$ $\quad V = \frac{1}{3} \ell w h$

Where $B = \ell w$ $\quad V = \frac{1}{\cancel{3}_1} \times 2 \times 2 \times \overset{1}{\cancel{3}}$

$$V = 4\text{m}^3$$

SELF-CORRECTING EXERCISE D, (continued) Pages 16 and 17.

2. V = ? π = 3.14 r = 8 cm h = 12 cm

Use $V = \frac{1}{3}Bh$

Where $B = \pi r^2$

$$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{\cancel{3}_1} \times 3.14 \times 8 \times 8 \times \cancel{12}^4$$
$$= 803.84 \text{ cm}^3$$

3. V = ? n = 5 a = 1.6 m b = 2.5 m h = 2 m

Use $V = \frac{1}{3}Bh$

Where $B = \frac{nab}{2}$

$$V = \frac{1}{3}\left(\frac{nab}{2}\right) \times h$$
$$= \frac{1}{3} \times \left(\frac{5 \times \cancel{1.6}^{0.3} \times 2.5}{\cancel{2}_1}\right) \times 2$$
$$= \frac{7.5}{3}$$
$$= 2.5 \text{ m}^3$$

4. V = 753.6 cm³ π = 3.14 r = ? h = 20 cm

Use $V = \frac{1}{3}Bh$

Where $B = \pi r^2$

$$V = \frac{1}{3}(\pi r^2)h$$
$$753.6 = \frac{1}{3} \times 3.14 \times r^2 \times 20$$
$$753.6 = \frac{62.8\,r^2}{3}$$
$$753.6 \times \frac{3}{62.8} = r^2$$
$$36 = r^2$$
$$6 = r$$

END OF LESSON 29

STUDENT'S COMMENTS

Since this is the last lesson in this course, we thought that you might like to tell us what you liked and disliked about it. There is a good reason for this. It will help us to see how this course, and others like it, can be improved in the future. You will be helping both yourself and us by answering the following questions about the course as honestly and accurately as you can.

1. Did you find the course useful? Why or why not?

2. Was there any section of the course that you especially liked? Why?

3. Was there any section of the course that you didn't like? Why?

4. Do you feel that there was enough information given to do all the lessons?

5. Do you feel that your teacher gave you enough help with this course?

6. Were there any sections of this course that you found too difficult? If so, which sections were they?

7. Were there any sections of this course that you found too easy? If so, which sections were they?

8. If you had a chance to take part in the writing of this course, what things would you do in order to improve it?

_____

_____

_____

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LESSON RECORD FORM

0902 Math 9
Revised 11/90

FOR STUDENT USE ONLY

Date Lesson Submitted

_____

Time Spent on Lesson

_____

(If label is missing or incorrect)

File Number

_____

Lesson Number _____

FOR A.D.L.C. USE ONLY

Assigned Teacher: _____

Lesson Grading: _____

Additional Grading E/R/P Code: _____

Mark: _____

Graded by: _____

Assignment Code: _____

Date Lesson Received:

_____

Lesson Recorded _____

Student's Questions and Comments

Apply Lesson Label Here

Name

Address

Postal Code

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

_____

Correspondence Teacher

St. Serv. 14-91

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do **not** enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

In this lesson you will review the concepts taught in Lessons 1 to 29.

PART A

Marks

8 1. Evaluate the following. Show all steps.

(a) $5^2 =$ (b) $(-9)^2 =$

(c) $8^{-2} =$ (d) $6(4)^2 =$

(e) $10^2 \times 10^3 =$ (f) $13^5 \div 13^3 =$

(g) $6^2 \times 2^3 =$ (h) $4^2 + (-5)^2 =$

4 2. Simplify the following.

(a) $(-2x^2y)(-6x^3y^2)$

(b) $\dfrac{3f^4g^3h^5}{f^3g^3h}$

Marks 12

3. Perform the following operations. Show your work.

(a) $1\frac{2}{5} + 1\frac{2}{9} =$

(b) $3\frac{1}{4} - 1\frac{3}{8} =$

(c) $2\frac{2}{9} \div 4\frac{2}{3} =$

(d) $4 - 3.68 =$

(e) $2.6 \times 0.034 =$

(f) $10 \div 0.11 =$

Marks 12

4. Solve the following equations. Show your solution steps.

(a) $7 - 2x = 9$

(b) $-\frac{3}{5}x = 18$

(c) $12x - 8x = 52$

(d) $5x - 12 = 4 - 3x$

(e) $\frac{x}{2} + \frac{x}{3} + \frac{x}{4} = -26$

(f) $\frac{3 - x}{5} - \frac{5 - 3x}{10} = \frac{1}{10}$

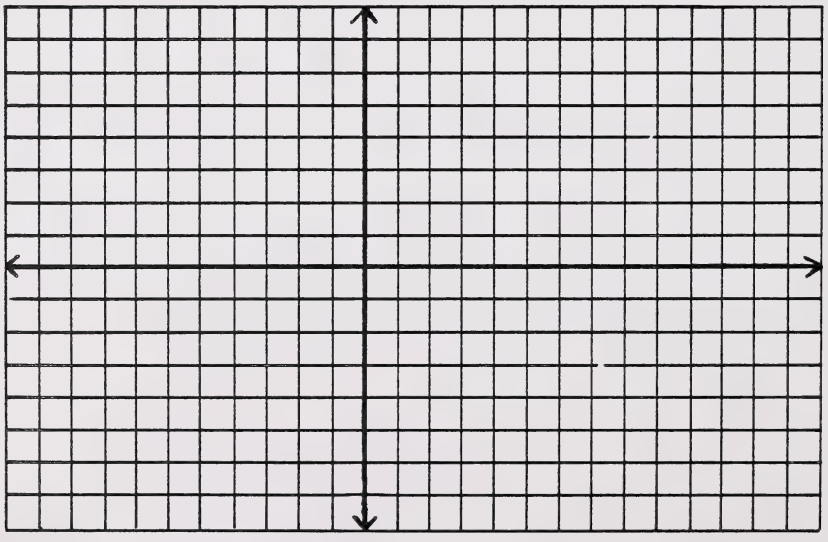
Marks
5

5. Three times a certain number is 42 more than the number itself. Find the number.

5

6. Fill in a table of values for the equation $y = 3x + 2$, $X \in R$. Then draw its graph.

x							
y							



Total for Part A _____ (maximum possible: 46 marks)

PART B

Marks

4 1. (a) Find the sum of $3x - 5y$, $5x - 3y$, and $2x - 2y$.

(b) Add $4x^2 - 3x + 1$ and $x^2 - 7x + 2$.

4 2. (a) Subtract $8x - 6y$ from $5x - 3y$.

(b) Subtract $7x^3 + 3x + 7$ from $6x^3 - 3x + 2$.

3 3. Simplify $(x^2 + 2x) - (x^2 - x) + (6x - 3x^2)$.

Marks

4. (a) Divide $-24x^2y + 8x^6y$ by $-8x^2y$.

4

(b) Divide $9x^3 - 12x^2 + 15x$ by $3x$.

4 5. (a) Multiply $-3xy$ by $-2x^2 - 3x + 4$.

(b) Multiply x^2y by $-2x^4y + xy^2$.

3 6. Simplify $3m - m(m - 3) - 6m^2$.

6 7. Multiply.

(a) $(3x - 7)(3x + 7) =$ _____

(b) $(2x - 5y)^2 =$ _____

(c) $(2a + 7b^2)(5a - 2b^2) =$ _____

Marks 15

8. Factor each of the following expressions completely.

(a) $x^2 - x - 30 =$

(b) $x^2 + 6xy + 9y^2 =$

(c) $a^2 - 100 =$

(d) $2ax^2 + 2axy =$

(e) $3x^2 - 12xy + 9y^2 =$

Marks 5

9. A metre of solid colour cloth costs $8.00. A metre of patterned cloth costs $12.00. Mrs. Walter bought three times as much patterned cloth as solid colour cloth. If the bill for the cloth was $220.00, how many metres of each type of cloth did she buy?

4

10. Fill in the blanks to make each statement true.

(a) A polynomial is a sum of _____.

(b) A polynomial with one term is called a _____.

(c) The sign on a term is included in the _____ coefficient.

(d) The degree of $2x^2y - 3x^4 + 2$ is _____.

Total for Part B _____ (maximum possible: 52 marks)

PART C

Marks

4 1. Randy's tax deduction last year was \$9 230. If his total earnings were \$36 920 what is the rate of deduction?

6 2. Keeping in mind the general sentence form "a is r% of b", solve the following.

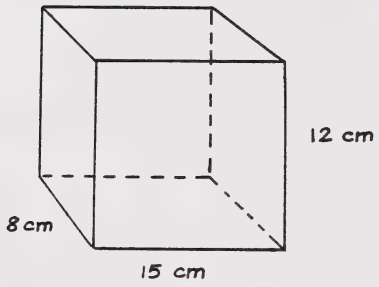
(a) 245 is 80% of what number?

(b) What percent of 160 is 480?

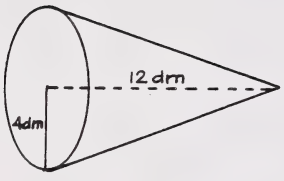
Marks 8

3. Find the volume and surface area of the following figures. Show all steps.

(a) Volume = Surface Area =



(b) Volume = Surface Area =



4

4. Remembering the rules for order of operations, do the following.

(a) $20 - 4 \times 4 + 15 \div 3 =$

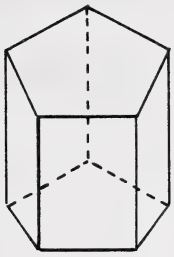
(b) $\dfrac{-24 - 8}{4 \times 3 - 4} =$

Marks

4

5. Name each of the following solids by the shape of its base.

(a)



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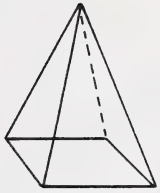
(b)



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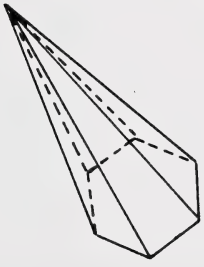
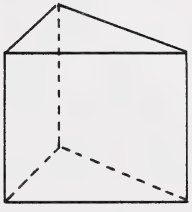
(c)



_____

(d)



_____

6

6. Write True or False beside the following statements.

_____ A hexagonal prism has a base that has six sides.

_____ A cube is a rectangular prism.

_____ Volume is always measured in square units.

_____ A cube has all eight sides of equal size.

_____ A pyramid is not a cone.

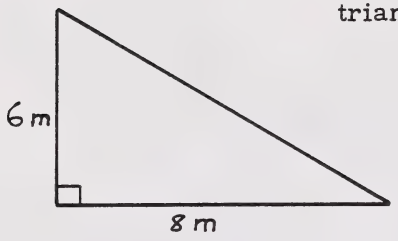
_____ The volume of a pyramid is one-third the volume of a cylinder.

Marks

4

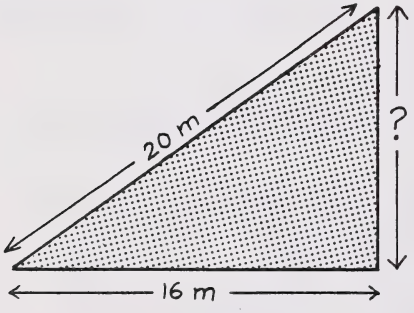
7. The formula for the Pythagorean Theorem is $c^2 = a^2 + b^2$. Do the following questions using this formula.

(a) Find the length of the hypotenuse of the following triangle.



4

(b) A ramp is 20 m long and its base is 16 m long. What would the height of the ramp be?



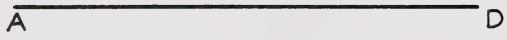
Marks

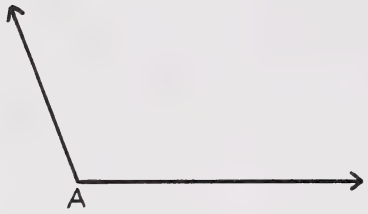
2

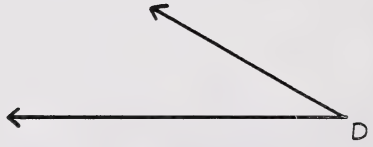
8. Bisect the following angle. Be sure to show all the construction arcs.

Marks
5

9. Given that $\triangle MAD$ is an ASA triangle. Construct it using only a compass and a straight edge.







Total for Part C _____ (maximum possible: 47 marks)

Total for Test _____ (maximum possible: 145 marks)

END OF COURSE

Well, you finally did it! You've finished the course.
Now all you have to do is write the final test.

N.L.C. - B.N.C.
3 3286 10936233 1